Coupled Adjoint-based Rotor Design using a Fluid Structure Interaction in Time Spectral Form

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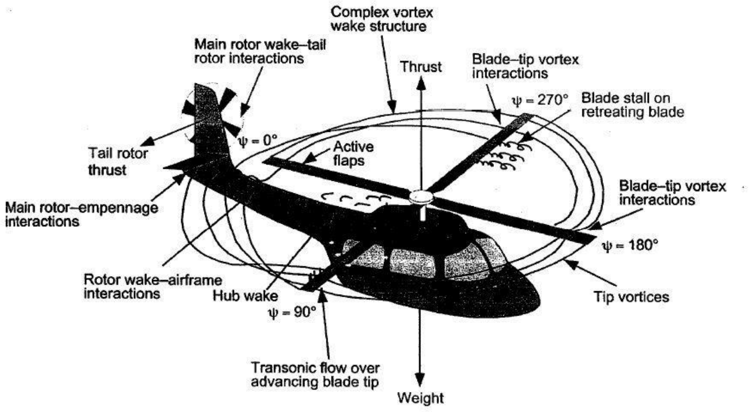
Introduction: Motivation

- Motivation
  - The system involving **multi-physics** is difficult to analyze with high fidelity analysis.
  - For example, difficulties associated with simulation of helicopter flight
    - **Aerodynamic challenges**: Complexity of unsteady and vortical air flows
    - **Structural dynamic** challenges: Long and flexible rotor blades
    - **Coupled physics** of aerodynamics and structures

- Objective
  - *Develop an accurate but efficient analysis and design method for rotor blade.*



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Source: https://en.wikipedia.org/wiki/File:UH-60_2nd_Squadron,_2nd_Cavalry_Regiment_(cropped).jpg

Introduction: Literature Review

- Literature Review for Rotor Design
  - A comprehensive rotor design code
    - CAMRAD (NASA Ames Research Center / U.S. Army, 1980)
      - Lifting-line theory, Lifting-surface theory
    - CAMRADII (Johnson Aeronautics,1994)
    - UMARC (University of Maryland, 1990)
      - FEM formulation + Quasi-steady 2-D strip theory
  - CFD/CA coupling
    - CAMRAD II, RCAS and UMARC started to include main rotor 3D-CFD coupling. (in 2000s)
    - Hybrid solver: OVERFLOW + CHARM (by CDI, 2016)

Partner	Partner label	CFD Code	CSD Code
U.S. Army Aero-flightdynamics Dir.	AFDD-1	OVERFLOW	CAMRADII
U.S. Army Aero-flightdynamics Dir.	AFDD-2	NSU3D-SAMARC	RCAS
NASA-Langley	NL-1	OVERFLOW	CAMRADII
NASA-Langley	NL-2	FUN3D	CAMRADII
Georgia Institute of Technology	GIT-1	FUN3D	DYMORE4
Georgia Institute of Technology	GIT-2	GENCAS	DYMORE2
Konkuk University	KU	KFLOW	CAMRADII
University of Maryland	UM	TURNS	UMARC
German Aerospace Center	DLR	N/A	S4

Smith, Marilyn J., et al. "An assessment of CFD/CSD prediction state-of-the-art using the HART II international workshop data." *68th Annual Forum of the American Helicopter Society, Ft. Worth, TX*. 2012.

Introduction: Contributions

▣ Literature Review

- Adjoint-based shape optimization for **static** aeroelastic problem
  - Kenway, Kennedy and Martins (U.Mich, 2014)

- **Time Accurate approach** with unsteady adjoint-based shape optimization of rotor (Dynamic FSI problem)
  - Mishra, Mani and Mavriplis (Wyoming University, 2014)

- **Contributions of th present study**
  - **Time-Spectral form** of fluid and structural equations of motion (steady form of governing equations)
  - Steady adjoint formulation for **unsteady**, dynamic problems in time-spectral form.
  - Fluid-structure interface (FSI) and **coupled adjoint solution**

1. Kenway, Gaetan KW, Graeme J. Kennedy, and Joaquim RRA Martins. "Scalable parallel approach for high-fidelity steady-state aeroelastic analysis and adjoint derivative computations." *AIAA journal* 52.5 (2014): 935-951.
2. Mishra, Asitav, et al. "Time-dependent adjoint-based aerodynamic shape optimization applied to helicopter rotors." *Rn* 3 (2014): 2.

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Traditional FSI Approach

▣ *Partitioned and Staggered Approach*

$$V\frac{\partial u}{\partial t} + R(u) = 0$$

Fluid:
Computational Fluid Dynamics
(CFD, R(W,x)=0)

Pressure Field →

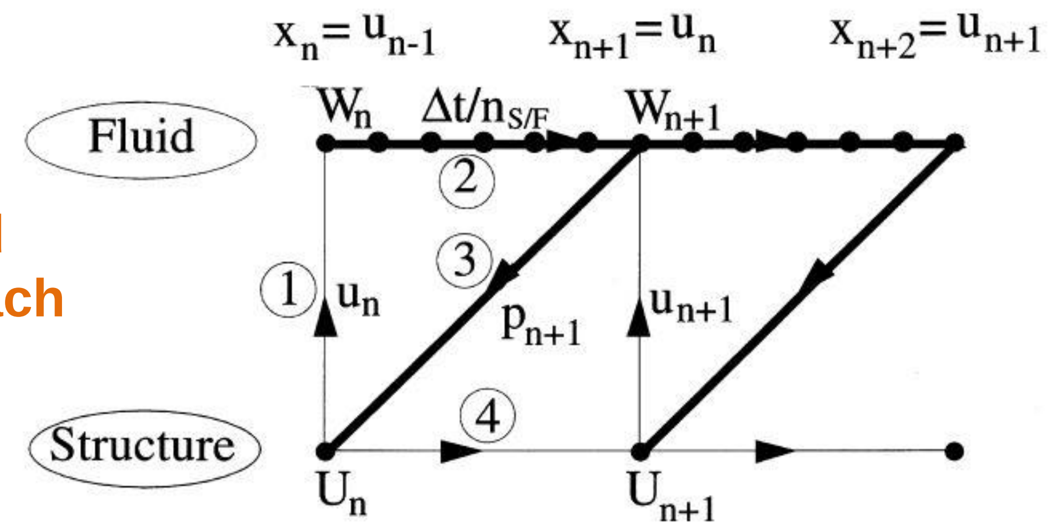
← Displacement Field

Structure:
Finite Element Method
(FEM, S(U,x)=0)

$$M\ddot{\boldsymbol{x}}(\psi) + C(\psi)\dot{\boldsymbol{x}}(\psi) + K(\psi)\boldsymbol{x}(\psi) = F(x,\dot{x},\psi)$$

- For the unsteady problems, physics of the problem limits the time step

serial staggered partitioned approach



W_n: fluid state vector
U_n: structural state vector
x_n: position of grid point
P_n: fluid pressure
u_n: structural displacement

Source: C. Farhat, M. Lesoinne, "Two efficient staggered algorithms for the serial and parallel solution of three-dimensional nonlinear transient aeroelastic problems", Computational Methods in Applied Mechanics and Engineering, Vol. 182, 2000, Pg. 499-515

Time Spectral Formulation (Fluids)

$$V\frac{\partial u}{\partial t}+R(u)=0$$

- For collocation method, the equation should be satisfied at each time instance (Fourier collocation point)

$$V\left.\frac{\partial u_j^N}{\partial t}+R(u_j^N)\right|_{t=t_j}=0 \quad (j=0,\ldots,N-1)$$

$$\begin{cases} u_j^N = \sum_{k=-N/2}^{N/2} \tilde{u}_k e^{ikt_j} & (j=0,\ldots,N-1),\ t_j=\frac{T}{N}j \\ \tilde{u}_k = \frac{1}{N}\sum_{j=0}^{N-1} u_j^N e^{-ikt_j} & (k=-N/2,\ldots,N/2-1) \end{cases}$$

- Fourier collocation derivative in (physical) time space

$$\frac{\partial u_l^N}{\partial t}=\sum_{k=-N/2}^{N/2-1} ik\tilde{u}_k e^{ikt_l} \quad (l=0,\ldots,N-1)$$

$$=\sum_{k=-N/2}^{N/2-1} e^{ikt_l}\frac{ik}{N}\sum_{j=0}^{N-1}u_j^N e^{-ikt_j} \quad (l=0,\ldots,N-1)$$

$$=\sum_{j=0}^{N-1}\left(\frac{1}{N}\sum_{k=-N/2}^{N/2-1} ike^{ik(t_j-t_l)}\right)u_j^N \quad (l=0,\ldots,N-1)$$

$$=\sum_{j=0}^{N-1}(D_N)_{lj}u_j^N \quad (l=0,\ldots,N-1)$$

, where $(D_N)_{lj}=\begin{cases}\frac{1}{2}(-1)^{l+j}\operatorname{cosec}\left(\frac{(l-j)\pi}{N}\right) & : l\neq 0 \\ 0 & : l=j\end{cases}$

$$D_t u^n = \begin{pmatrix} 0 & d_1 & \cdots & d_{\frac{N-1}{2}} & -d_{\frac{N-1}{2}} & \cdots & -d_1 \\ -d_1 & 0 & d_1 & d_2 & \cdots & \cdots & -d_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ d_1 & d_2 & \cdots & \cdots & -d_2 & -d_1 & 0 \end{pmatrix}\begin{pmatrix} u^1 \\ u^2 \\ \vdots \\ u^N \end{pmatrix}$$

- Final form of the time-spectral equation in a time-domain and steady state !

$$VD_N U+R(U)=0,$$ where $U=(u_0^N,u_1^N,\ldots,u_{N-1}^N)$

$$V\frac{\partial u_j^N}{\partial t}+VD_N u_j^N+R(u_j^N)=0 \quad (j=0,\ldots,N-1)$$

Time Spectral Formulation (Structures)

- Dynamic Structural Analysis

  - Governing equation

$$M\ddot{\boldsymbol{x}}(\psi) + C(\psi)\dot{\boldsymbol{x}}(\psi) + K(\psi)\boldsymbol{x}(\psi) = F(x, \dot{x}, \psi)$$

  - First order ODE form

$\boldsymbol{x}$: nodal displacements
$\boldsymbol{\psi}$: azimuth angle (0 ° ~ 360°)

$$\dot{y}(\psi) = Ay(\psi) + Bf(\psi)$$

where, $y(\psi) = \begin{bmatrix} x(\psi) \\ \dot{x}(\psi) \end{bmatrix}$, $f(\psi) = \begin{Bmatrix} 0 \\ F \end{Bmatrix}$, $A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}$, and $B = \begin{Bmatrix} 0 \\ M^{-1} \end{Bmatrix}$

Time Spectral Formulation (Structures)

- Dynamic Structural Analysis using Spectral Formulation
  - State-vector form equation

$$\dot{y}(\psi) = Ay(\psi) + Bf(\psi)$$

  - Assuming the solution with a Fourier series

$$y(t) = \widehat{y_0} + \sum_{n=1}^{N_H} (\widehat{y_{cn}} \cos \omega nt + \widehat{y_{sn}} \sin \omega nt)$$

  - The time derivative can be converted into a Matrix Vector product. It can be solved by applying pseudo-time stepping

$$\frac{\partial y_{TS}}{\partial \tau} + D_N y_{TS} - A y_{TS} - \mathrm{B} f_{TS} = 0$$

τ: pseudo time

where, $y_{TS} = \begin{pmatrix} y(\psi_0 + \Delta\psi) \\ y(\psi_0 + 2\Delta\psi) \\ \vdots \\ y(\psi_0 + 2\pi) \end{pmatrix}$ $\quad f_{TS} = \begin{pmatrix} f(\psi_0 + \Delta\psi) \\ f(\psi_0 + 2\Delta\psi) \\ \vdots \\ f(\psi_0 + 2\pi) \end{pmatrix}$

Spectral Method

❖ **solution approximation**	❖ **error minimization**
- Trigonometric (Fourier)	- Galerkin
- Chebychev	- **Collocation**
- Legendre	- Tau

Proposed FSI in Time-Spectral Formulation

- Time Spectral form of Governing Equations

Pressure Field

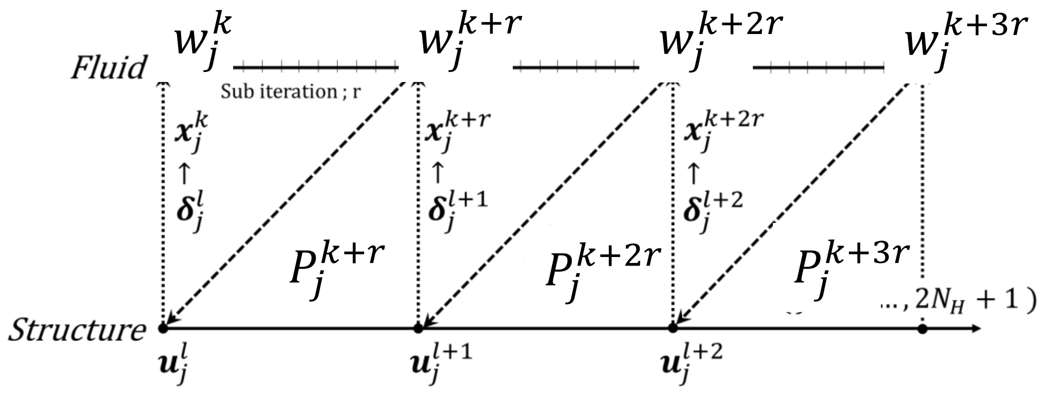
Displacement Field

Fluid Equation

$$V\frac{\partial \boldsymbol{w_{ts}}}{\partial \tau} + V\omega \boldsymbol{D} \boldsymbol{w_{ts}} + \boldsymbol{R}(\boldsymbol{w_{ts}}) = 0$$

Structural Equation

$$\frac{\partial \boldsymbol{y_{ts}}}{\partial \tau} + \omega \boldsymbol{D} \boldsymbol{y_{ts}} - \boldsymbol{A} \boldsymbol{y_{ts}} - \boldsymbol{B} \boldsymbol{f_{ts}} = 0$$



Time-spectral Procedure – Pseudo-time level

w_j^k: fluid state vector

u_j^l: structural state vector

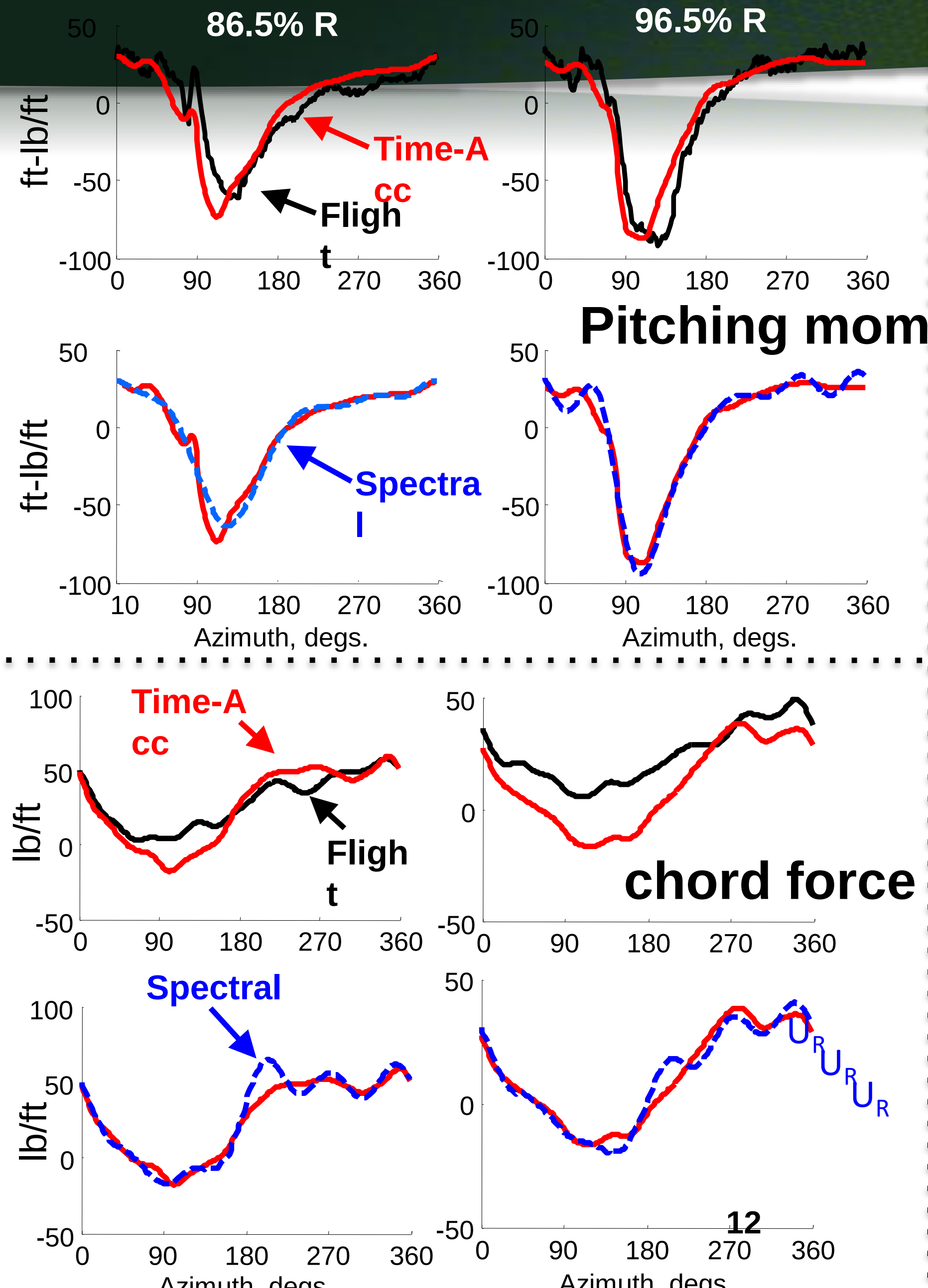
x_j^k: position of grid point

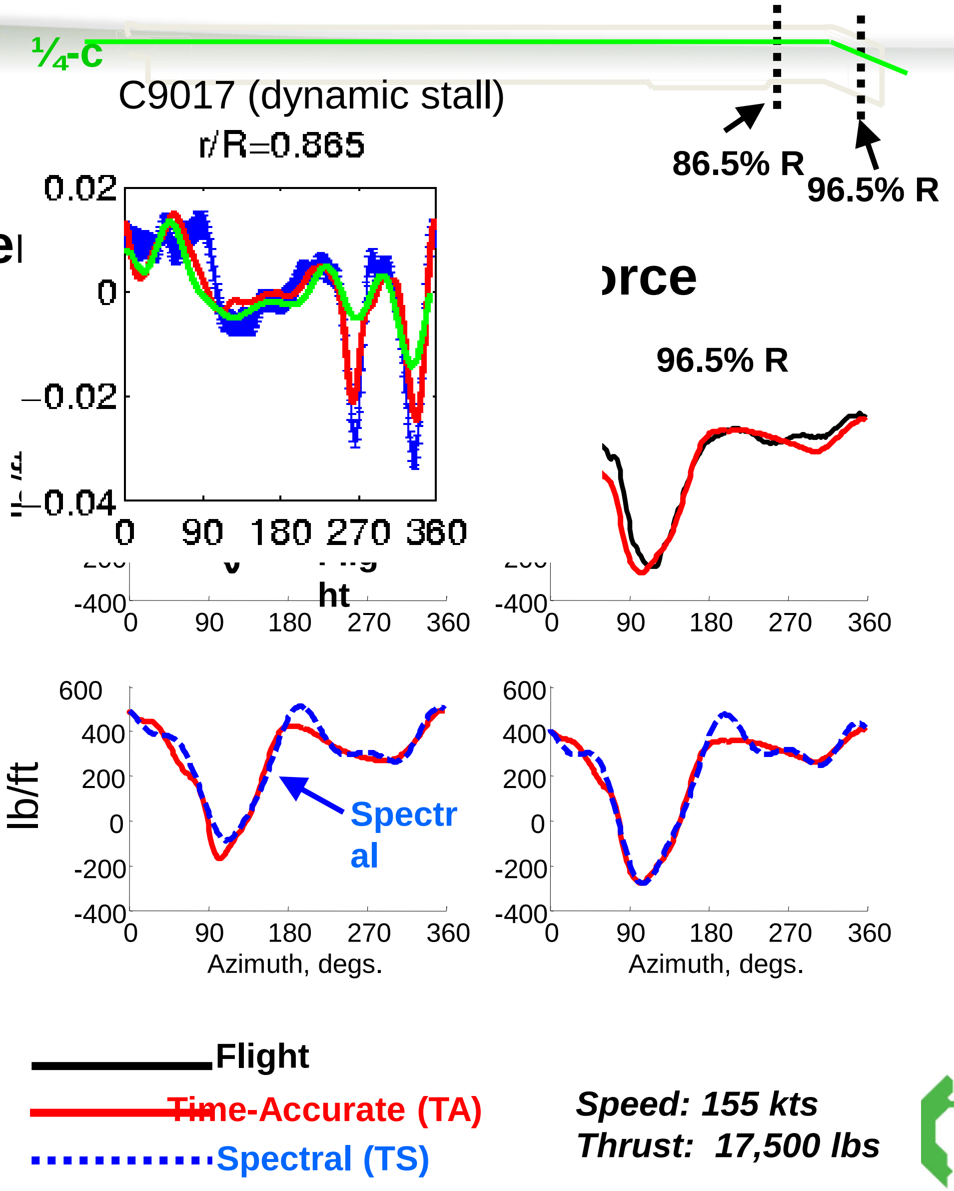
P_j^k: fluid pressure

δ_j^k: structural displacement

TS FSI Validation: Sectional Airloads Comparison (CFD/CSD)

UH-60A
C8534
with 7 harmonics





Speed: 155 kts
Thrust: 17,500 lbs

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Sensitivities and Design

- Approach: **Sensitivity Analysis for unsteady problems.**
  - Shape Optimization requires gradient with respect to a large number of design variables.

Finite Difference Method (FDM) → With N design variables
N+1 Aeroelastic Simulations

$$f'(x) \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

  - Adjoint Method is extremely handy in such cases

Adjoint Method → 1 Aeroelastic Solution + 1 Adjoint Solution

$$\left(\frac{\partial R}{\partial w}\right)^T \lambda = -\frac{\partial I}{\partial w}$$

  - Disadvantage: For unsteady problems, physics residuals and adjoint variables need to be stored at each physical time step→ Memory and time intensive.

Coupled Adjoint for Coupled Sensitivity Analysis

Coupled Adjoint Analysis

- Gradient computation using adjoint method for a coupled FSI problem
- Objective function: $I = I(w, y, x, b)$

w: fluid state variables
y: structural state variables
x: mesh state variables
b: design variables

- Total derivative of objective function w.r.t design variables

$$\frac{dI}{db} = \frac{\partial I}{\partial b} + \frac{\partial I}{\partial w}\frac{\partial w}{\partial b} + \frac{\partial I}{\partial y}\frac{\partial y}{\partial b} + \frac{\partial I}{\partial x}\frac{\partial x}{\partial b}$$

- State equations for fluid and structural system and their derivatives w.r.t design variables

Fluid: $R(w, y, x, b) = 0 \rightarrow \frac{dR}{db} = \frac{\partial R}{\partial b} + \frac{\partial R}{\partial w}\frac{\partial w}{\partial b} + \frac{\partial R}{\partial y}\frac{\partial y}{\partial b} + \frac{\partial R}{\partial x}\frac{\partial x}{\partial b} = 0$

Structure: $S(w, y, x, b) = 0 \rightarrow \frac{dS}{db} = \frac{\partial S}{\partial b} + \frac{\partial S}{\partial w}\frac{\partial w}{\partial b} + \frac{\partial S}{\partial y}\frac{\partial y}{\partial b} + \frac{\partial S}{\partial x}\frac{\partial x}{\partial b} = 0$

- As the residual derivatives are zero, they can be multiplied with an adjoint vector and added to the objective function derivative.

$$\frac{dI}{db} = \frac{\partial I}{\partial b} + \frac{\partial I}{\partial w}\frac{\partial w}{\partial b} + \frac{\partial I}{\partial y}\frac{\partial y}{\partial b} + \frac{\partial I}{\partial x}\frac{\partial x}{\partial b} + \lambda^T\left(\frac{\partial R}{\partial b} + \frac{\partial R}{\partial w}\frac{\partial w}{\partial b} + \frac{\partial R}{\partial y}\frac{\partial y}{\partial b} + \frac{\partial R}{\partial x}\frac{\partial x}{\partial b}\right) + \phi^T\left(\frac{\partial S}{\partial b} + \frac{\partial S}{\partial w}\frac{\partial w}{\partial b} + \frac{\partial S}{\partial y}\frac{\partial y}{\partial b} + \frac{\partial S}{\partial x}\frac{\partial x}{\partial b}\right)$$

= Zero   = Zero

λ: adjoint vector for flow state
ϕ: adjoint vector for structural state

Coupled Adjoint for Coupled Sensitivity Analysis

- Coupled Adjoint Analysis
  - Rearranging the terms

R : fluid residuals
S : Structural residuals
w: fluid state variables
y: structural state variables
x: mesh state variables
b: design variables

$$\frac{dI}{db} = \frac{\partial I}{\partial b} + \lambda^T \frac{\partial R}{\partial b} + \phi^T \frac{\partial S}{\partial b}$$

$$+ \left[\frac{\partial I}{\partial w} + \lambda^T \frac{\partial R}{\partial w} + \phi^T \frac{\partial S}{\partial w}\right]\frac{\partial w}{\partial b} + \left[\left(\frac{\partial I}{\partial y} + \lambda^T \frac{\partial R}{\partial y} + \phi^T \frac{\partial S}{\partial y}\right)\right]\frac{\partial y}{\partial b}$$

= Zero   = Zer
e

  - The arbitrary adjoint vector can be chosen to be independent of the derivative of state variables w.r.t. the design variables.
  - The adjoint equation to be solved

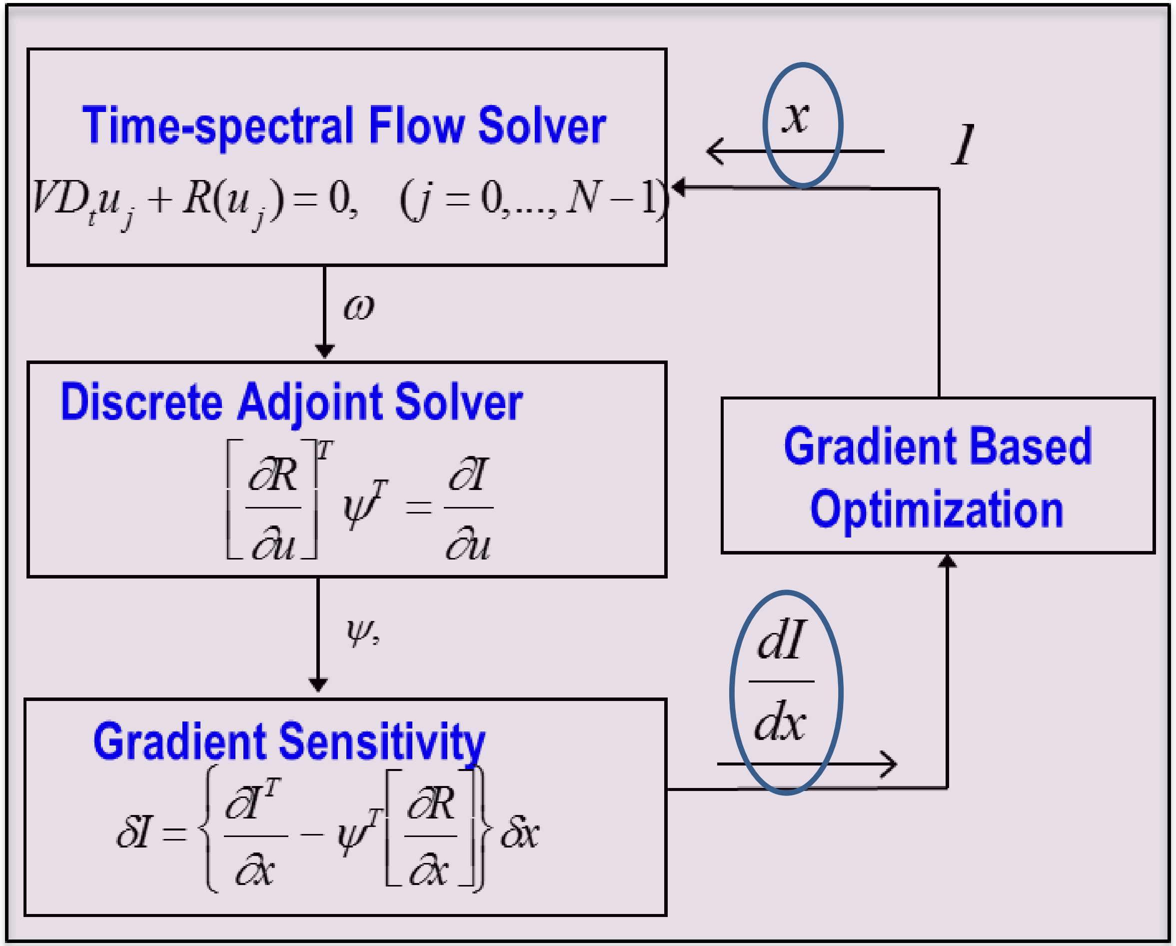
$$\begin{bmatrix} \frac{\partial R}{\partial w} & \frac{\partial R}{\partial y} \\ \frac{\partial S}{\partial w} & \frac{\partial S}{\partial y} \end{bmatrix}^T \begin{Bmatrix} \lambda \\ \phi \end{Bmatrix} = -\begin{bmatrix} \frac{\partial I}{\partial w} \\ \frac{\partial I}{\partial y} \end{bmatrix}$$

  - Total derivative of objective function w.r.t. design variables is

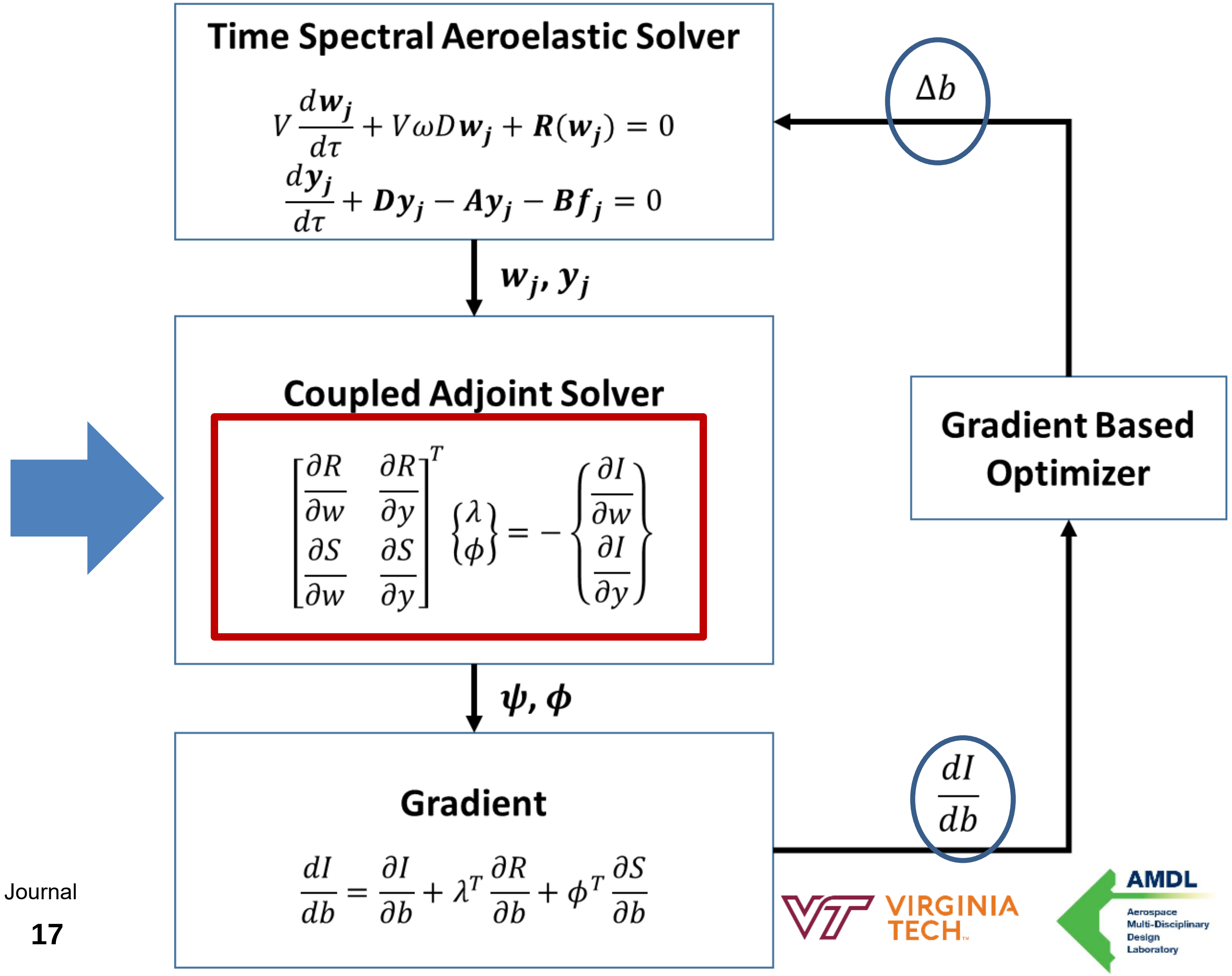
$$\frac{dI}{db} = \frac{\partial I}{\partial b} + \frac{\partial I}{\partial w}\frac{\partial w}{\partial b} + \frac{\partial I}{\partial y}\frac{\partial y}{\partial b} = \frac{\partial I}{\partial b} + \lambda^T \frac{\partial R}{\partial b} + \phi^T \frac{\partial S}{\partial b}$$

Design Framework

Aerodynamic Optimization



Aero-Structural Optimization



Source: S. Choi, K. Lee, J.J. Alonso "Helicopter Rotor Design using a Time Spectral and Adjoint Based Method", Journal of Aircraft, Vol. 51, No. 2, March-April, 2014,

Aero-Only Design at Forward Flight (Flight 8534 of UH-60A)

Design condition:

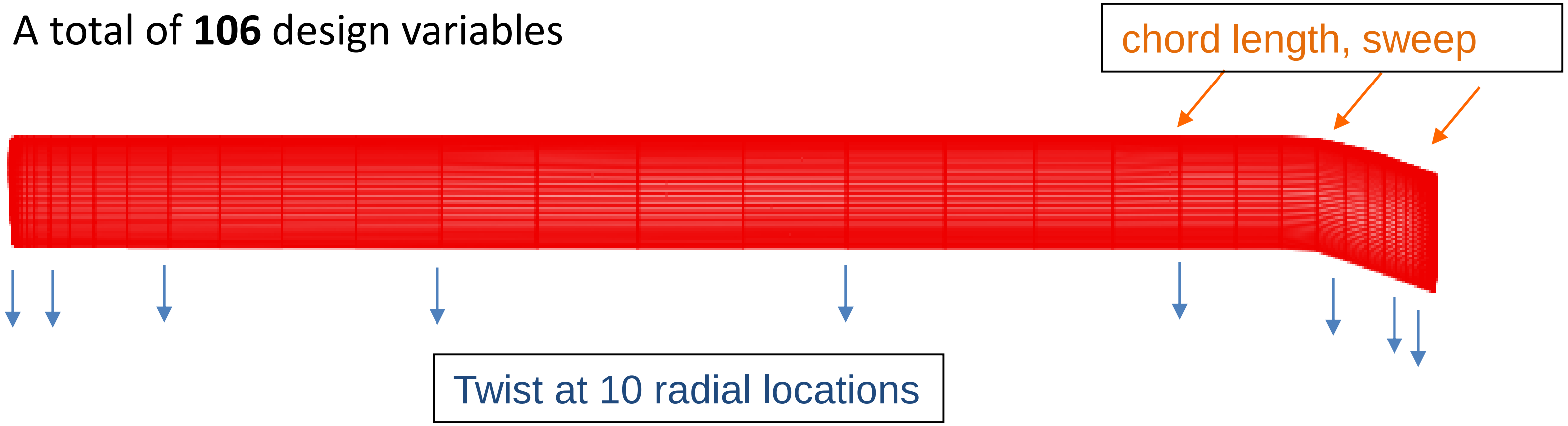
- At each iteration, new control angles and aeroelastic deformation should be provided from the Comprehensive Analysis
- Simplifications for "aerodynamic design"
  - constant aeroelastic deformation, and constant shaft angle.
  - only two constraints.

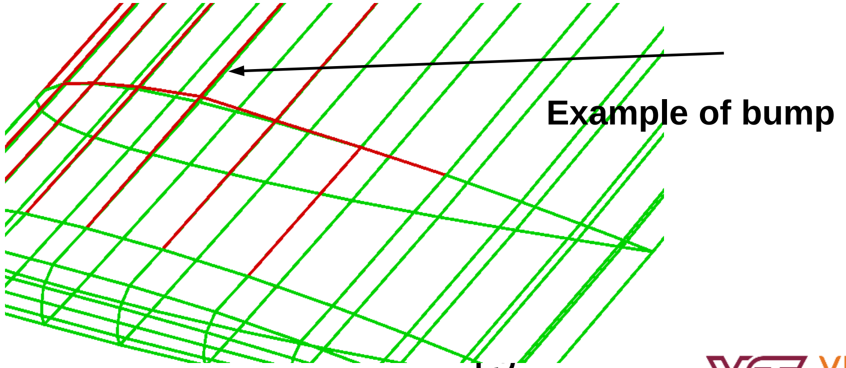
- Objective function : reduce Torque (C_Q).
- Constraint 1 : same Thrust (C_T).
- Constraint 2 : same or less Drag Force (C_D).

- A total of 4 harmonics (9 time instances) are used
- Using NPSOL (Nonlinear Programming SOLver).

Design variables

- Chord length and position of leading edge / sweep at 84.7% and 94.2%, 100% = 6
- Twist angle at the 10 span locations = 10
- Airfoil camber/thickness changes at 10 locations around airfoil along 9 sections on the span (using Hicks-Henne bump functions) = 90
- A total of **106** design variables





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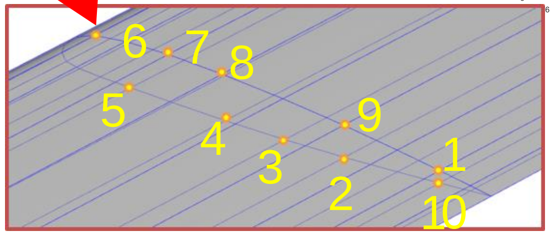
Sensitivity Analysis Results

- Sensitivity Analysis Results (Aero only)
  - Object Function : Drag

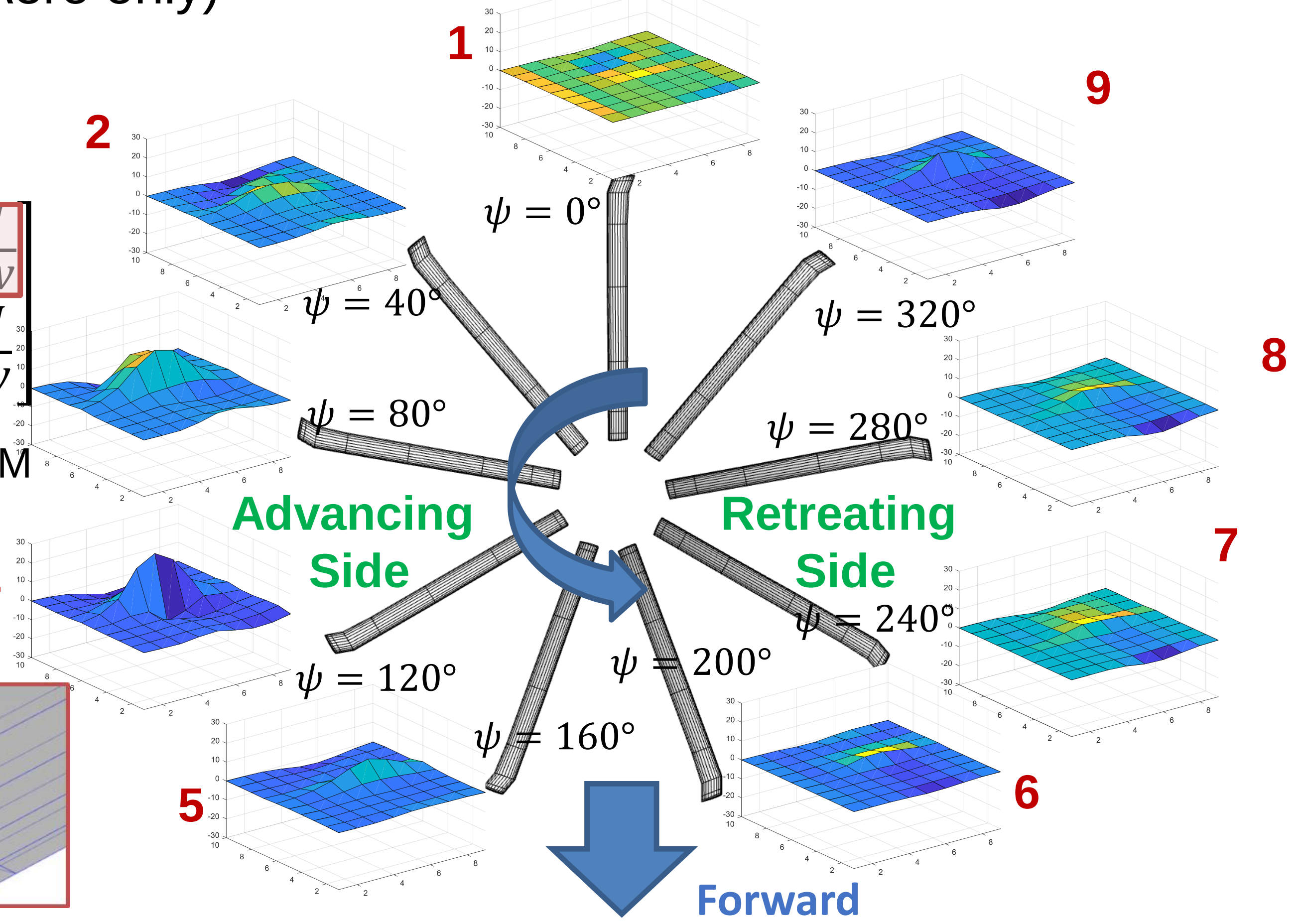
$$\begin{bmatrix} \frac{\partial R}{\partial w} & \frac{\partial R}{\partial y} \\ \frac{\partial S}{\partial w} & \frac{\partial S}{\partial y} \end{bmatrix}^T \begin{Bmatrix} \lambda \\ \phi \end{Bmatrix} = - \begin{bmatrix} \frac{\partial I}{\partial w} \\ \frac{\partial I}{\partial y} \end{bmatrix}$$

$\frac{\partial R}{\partial w}$: (368,640 x 368,640) Matrix

most sensitive location



$\frac{\partial I}{\partial b}(Drag)$



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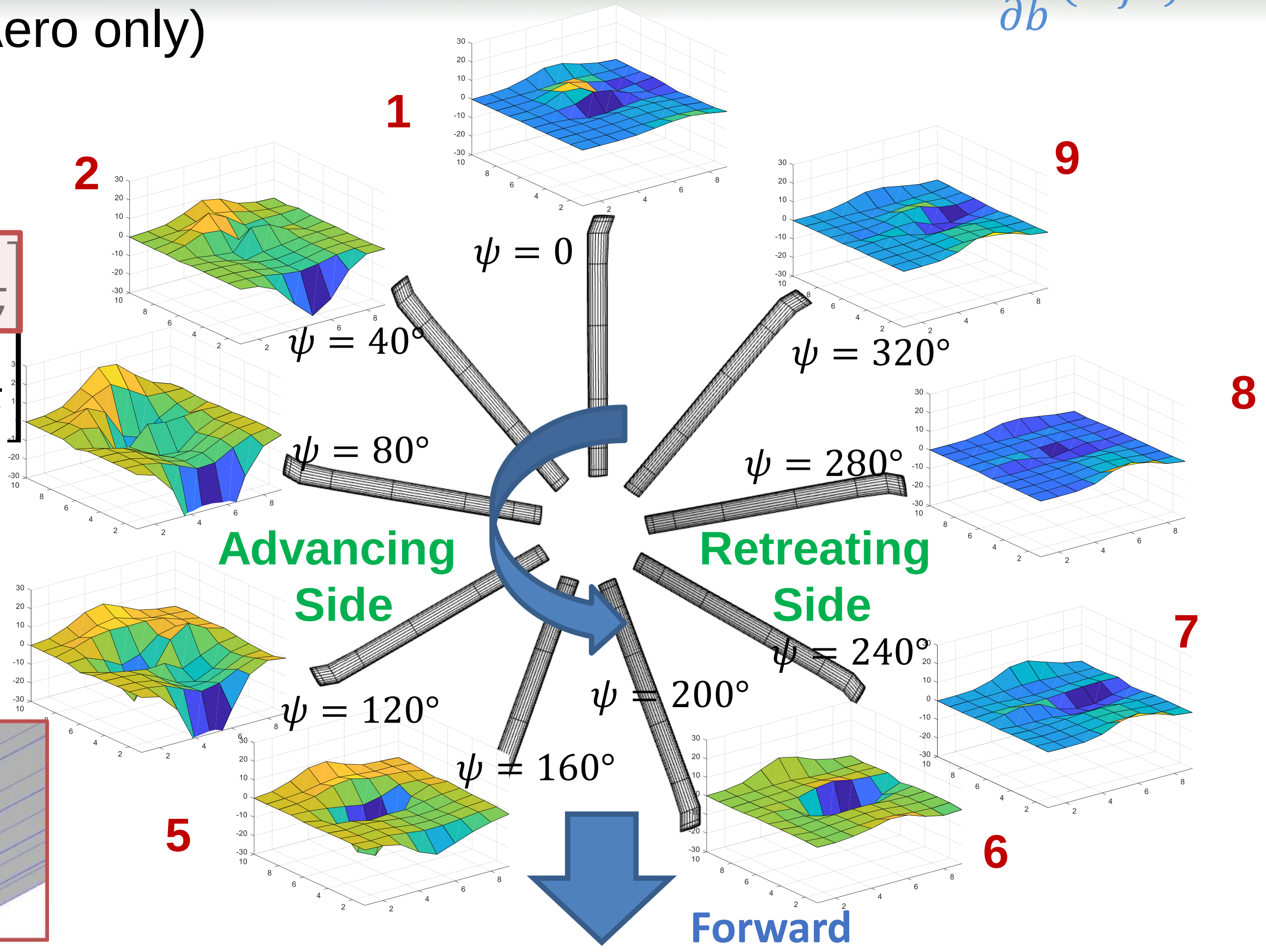
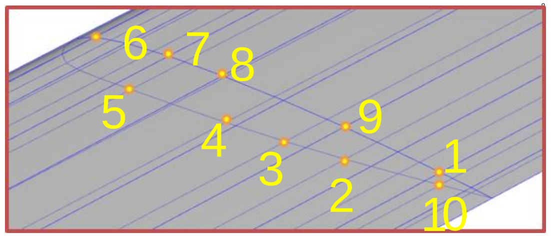
Sensitivity Analysis Results

- Sensitivity Analysis Results (Aero only)
  - Object Function : Lift

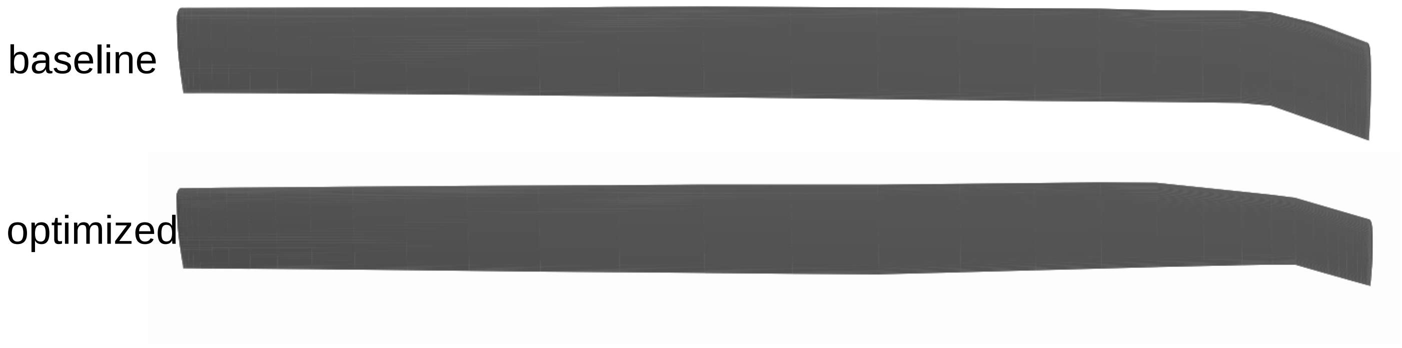
$$\begin{bmatrix} \frac{\partial R}{\partial w} & \frac{\partial R}{\partial y} \\ \frac{\partial S}{\partial w} & \frac{\partial S}{\partial y} \end{bmatrix}^T \begin{Bmatrix} \lambda \\ \phi \end{Bmatrix} = - \begin{bmatrix} \frac{\partial I}{\partial w} \\ \frac{\partial I}{\partial y} \end{bmatrix}$$

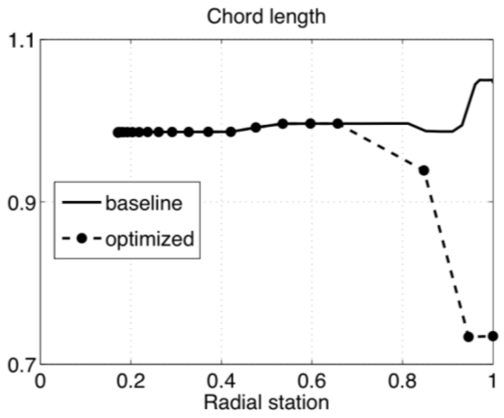
$\frac{\partial I}{\partial b}(Lift)$

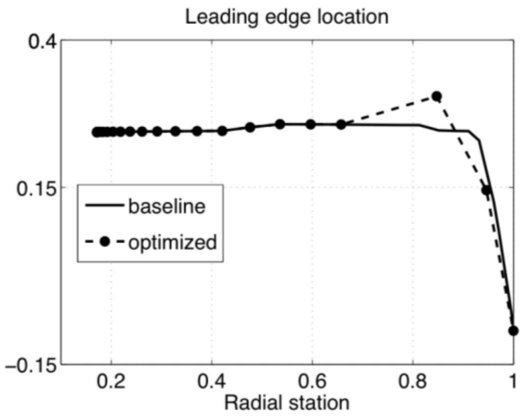


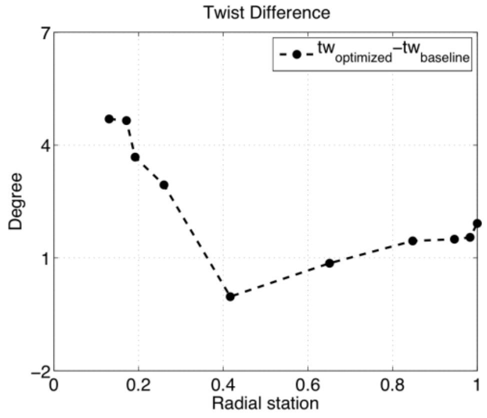


Optimized Results









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CFD/CSD Coupled Validation

- After 5 CFD/CSD coupling iterations.

CFD : Time-accurate, Navier-Stokes computation

CSD : UMARC (3 degree trim - thrust, rolling, pitching M constrained, shaft angle fixed)

Aero-optimization validation

7.4% reduction in torque.

almost constant in thrust.

but 30% increase in rolling M.

15% increase in pitching M.

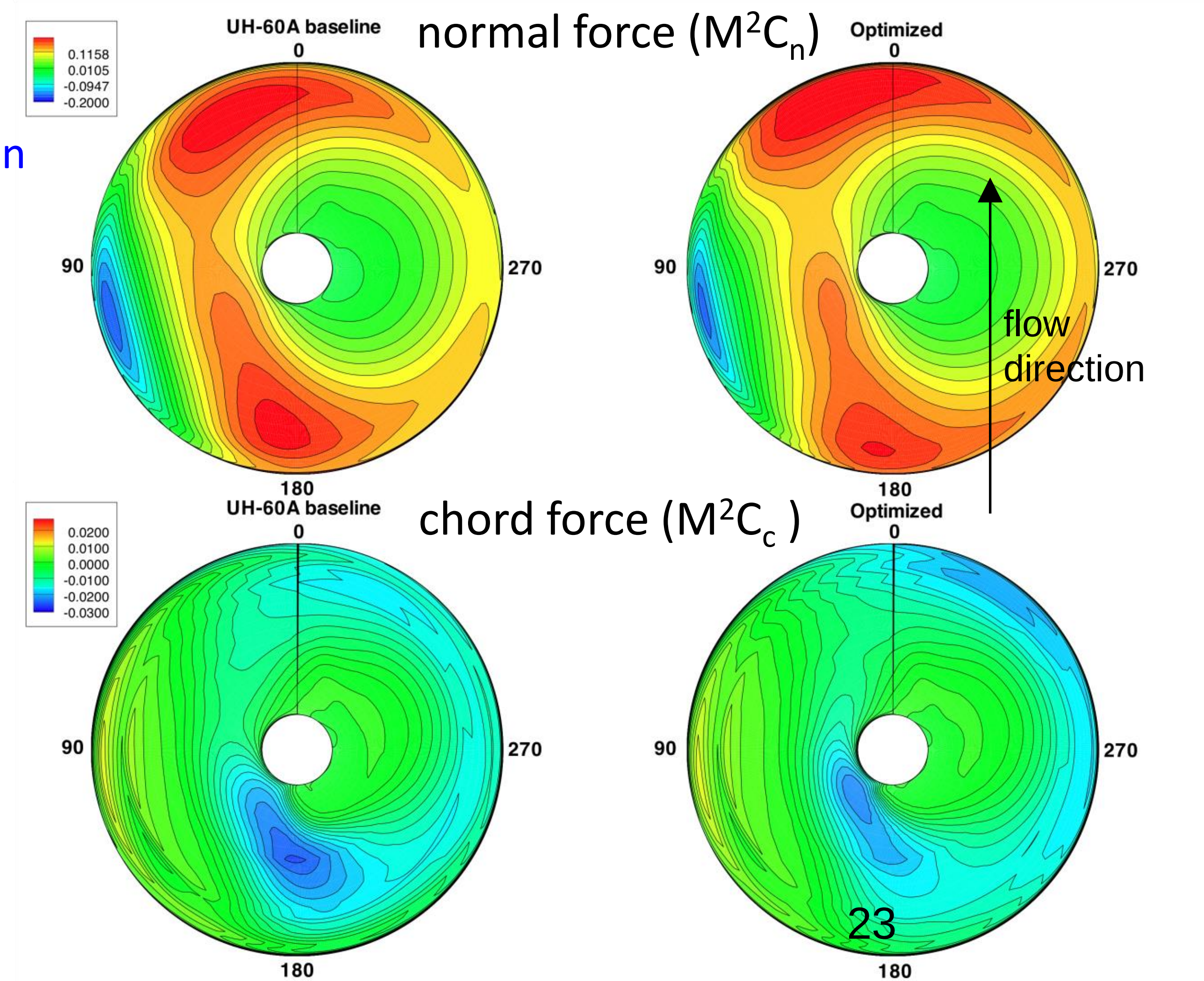
CFD/CSD coupled

5% decrease in torque.

Constant in thrust.

3% increase in rolling M.

2% increase in pitching M.



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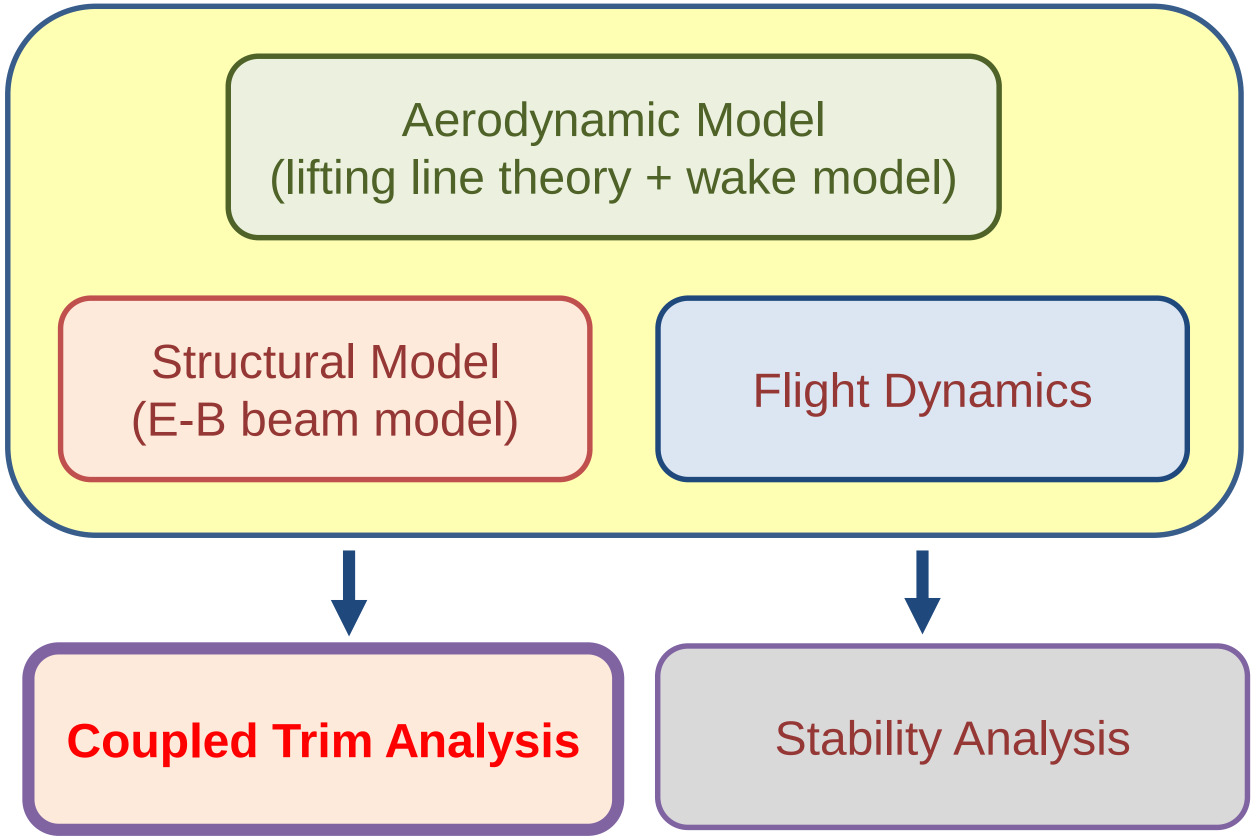
Computational Modelling: Comprehensive Analysis (CA)

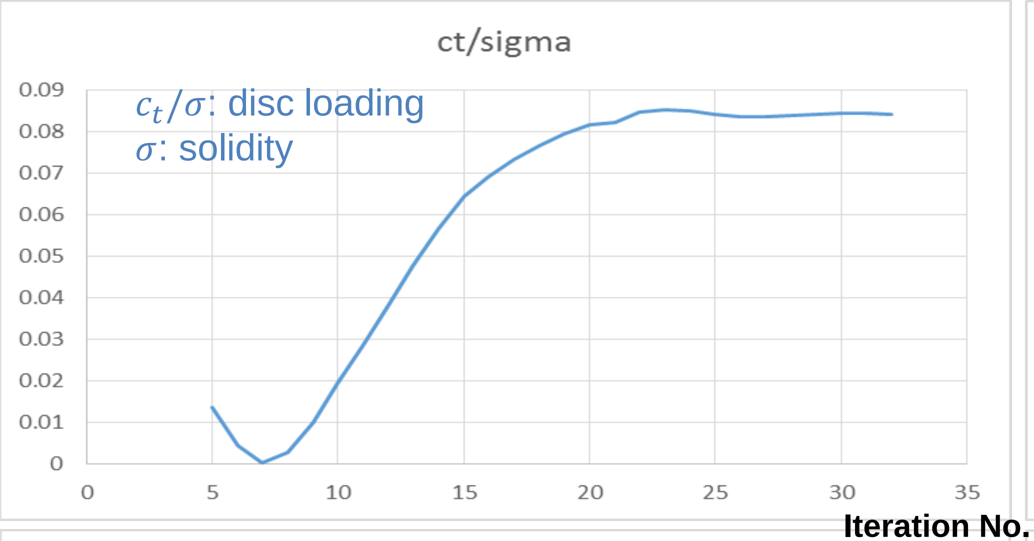
- UMARC : University of Maryland Advanced Rotorcraft Code [Ref.6]

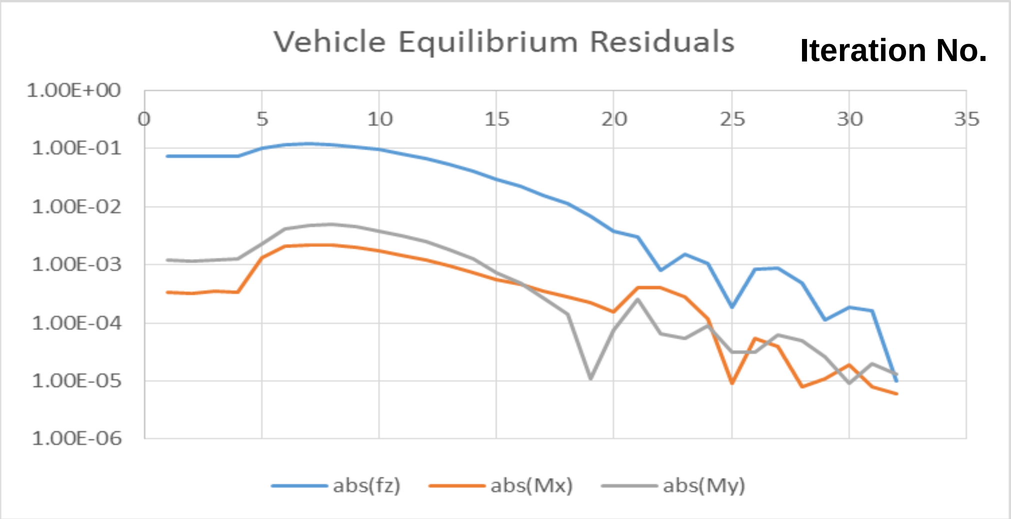
UMARC (CA)

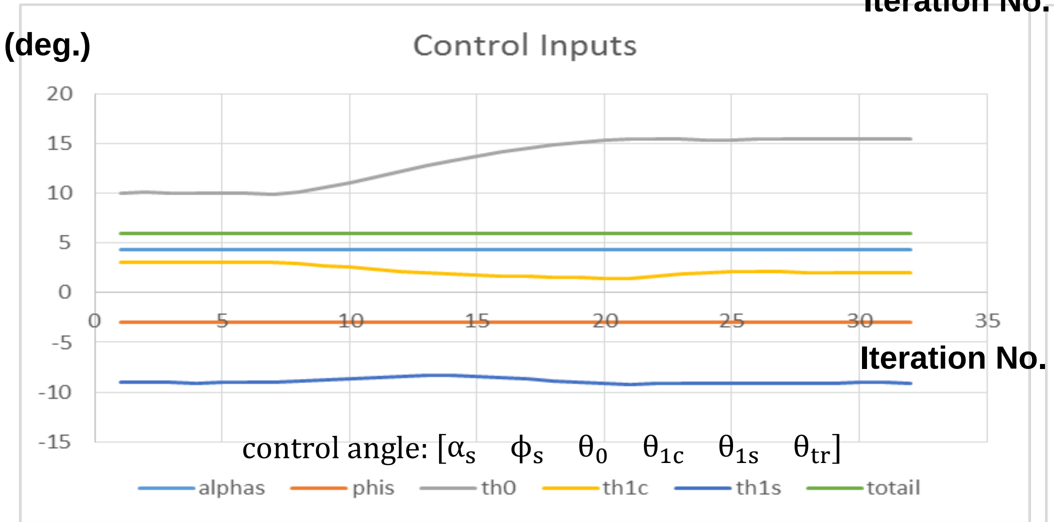


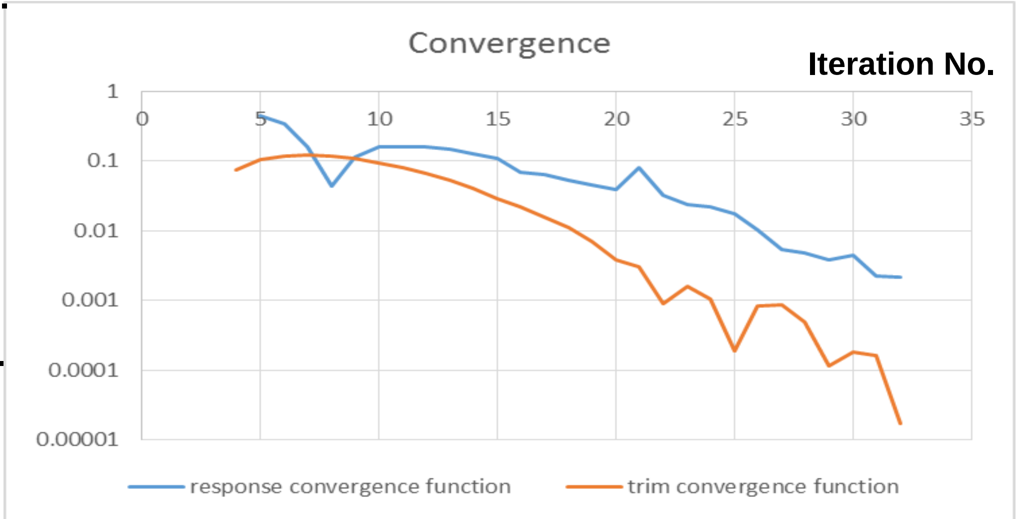
Computational Modelling: Structures

- Coupled Trim Analysis Results (UMARC)



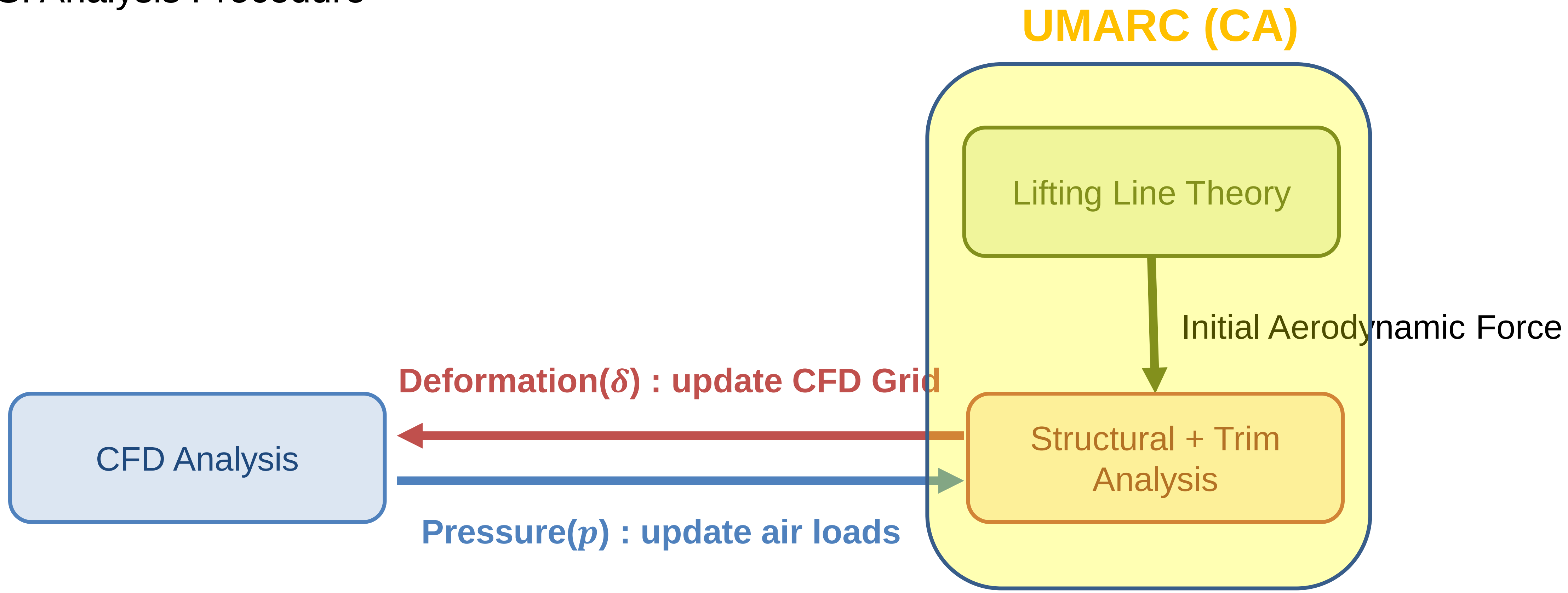






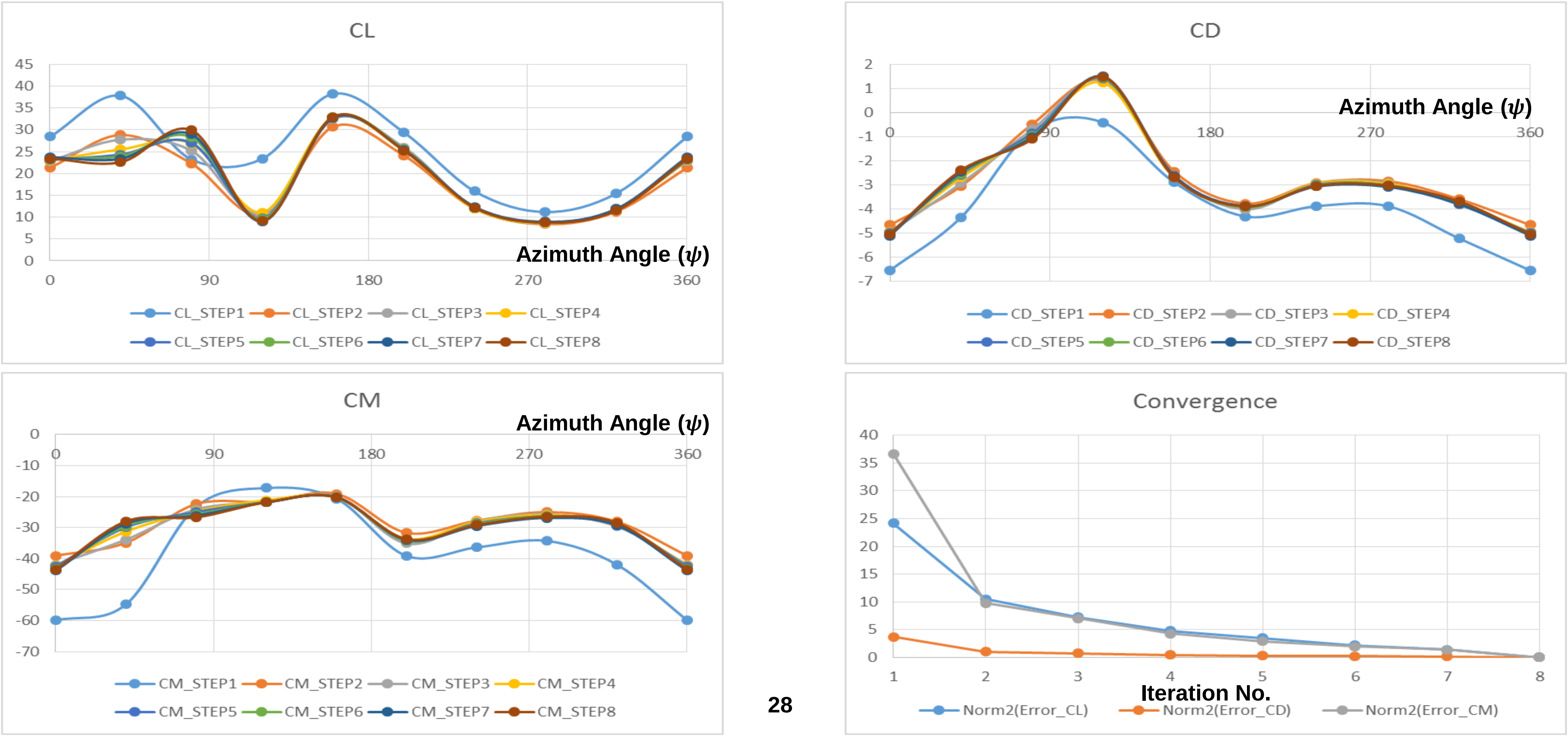
Computational Modelling: Loosely Coupled FSI Analysis

- FSI Analysis Procedure



Computational Modelling: Loosely Coupled FSI Analysis

▣ FSI Analysis Results



Sensitivity Analysis Results

▣ Coupled Sensitivity Analysis: Aero + Structure

Aerodynamic Jacobian

(368,640) (3,348)

(368,640)

(3,348)

$$\begin{bmatrix} \frac{\partial R}{\partial w} & \frac{\partial R}{\partial y} \\ \frac{\partial S}{\partial w} & \frac{\partial S}{\partial y} \end{bmatrix}^T \begin{Bmatrix} \lambda \\ \phi \end{Bmatrix} = -\begin{bmatrix} \frac{\partial I}{\partial w} \\ \frac{\partial I}{\partial y} \end{bmatrix}$$

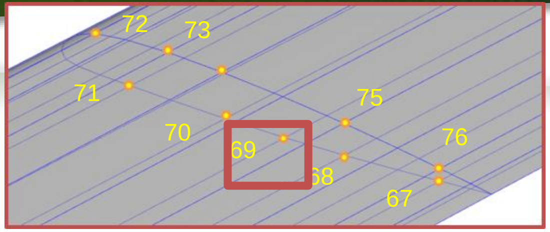
Coupled Cross Jacobians **Structural Jacobian**

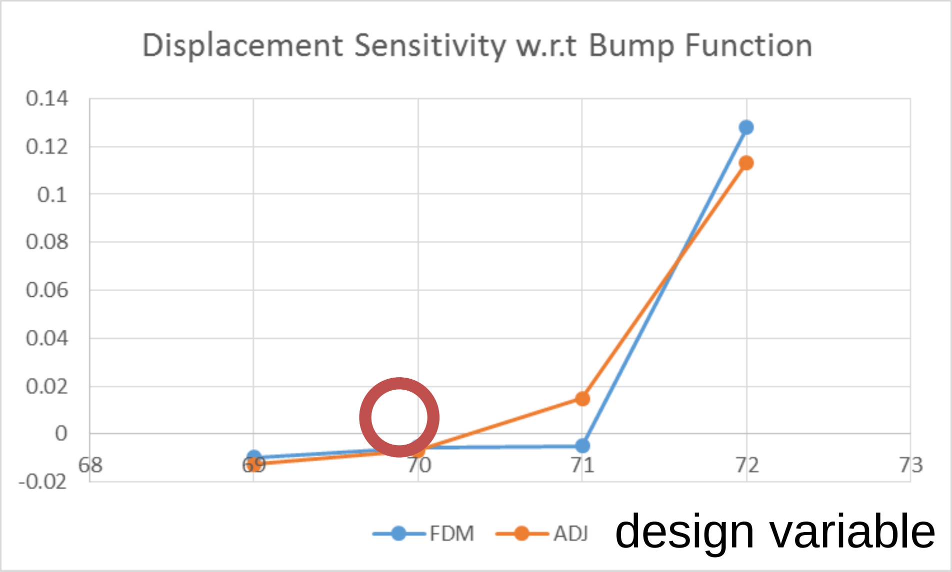
- ✓ $\frac{\partial R}{\partial w}, \frac{\partial S}{\partial y}$ and $\frac{\partial S}{\partial w}$ are sparse matrices, $\frac{\partial R}{\partial y}$ is densely populated matrix
- ✓ Due to the large size of the adjoint matrix, GMRES, a Krylov subspace solver has been used to solve the system.
- ✓ This has been implemented using PETSC, a suite of scalable and parallel routines for the solution of large scale PDEs.
- ✓ The above system takes around ~1500 iterations with 600 restart iterations to converge

Sensitivity Analysis Results

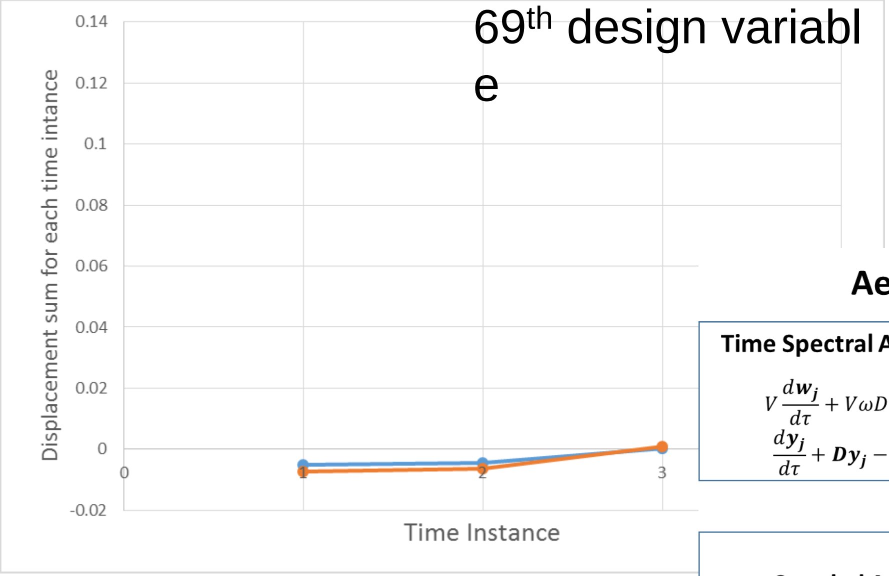
- Coupled Sensitivity Analysis: Aero + Structure

  I (objective function) = Sum of all displacement

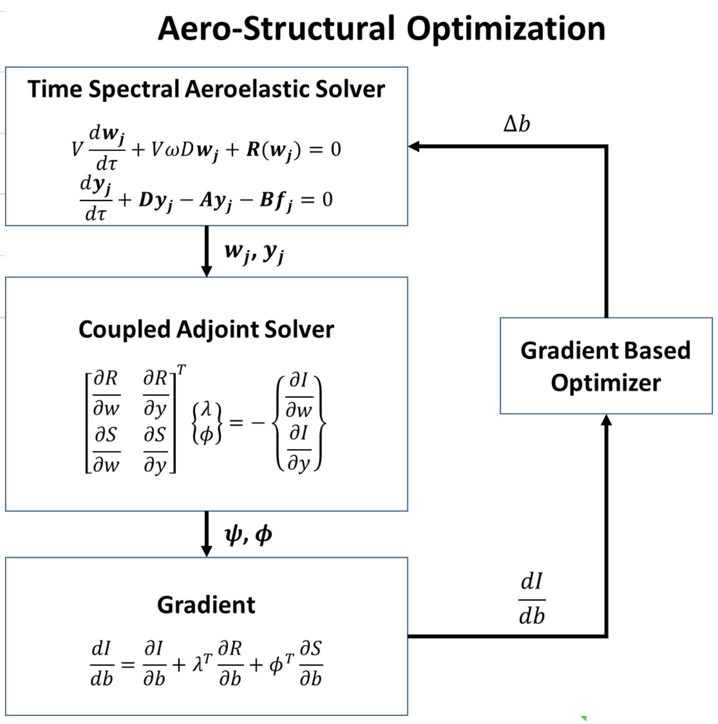




design variable N



69th design variable



- Future Design

  I (objective function) = torque

  C (constraints) = moments and thrust

Contents

I. Introduction

II. Fluid-Structure Interaction in Time Spectral Form

III. Adjoint Sensitivity for Time Spectral Form

IV. Sensitivity Analysis Results

I. Conclusions

Conclusion

- Time spectral and adjoint-based method is effective for the design involving multiphysics problems.
- An accurate but efficient coupled sensitivity analysis method for rotor design is developed.
- Aerodynamics only and Coupled sensitivity analysis is performed and validated by comparing with FDM results.
- Fluid-Structure coupled adjoint-based sensitivity analysis will be used to optimize the shape of rotor blade.

Thank you for your attention !

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Adjoint based Sensitivity Analysis

Coupled Adjoint Analysis

- Gradient computation using adjoint method for a coupled FSI problem
- Objective function: $I = I(w, y, x, b)$

$\boldsymbol{w}$: fluid state variables
$\boldsymbol{y}$: structural state variables
$\boldsymbol{x}$: mesh state variables
$\boldsymbol{b}$: design variables

- Total derivative of objective function w.r.t design variables

$$\frac{dI}{db} = \frac{\partial I}{\partial b} + \frac{\partial I}{\partial w}\frac{\partial w}{\partial b} + \frac{\partial I}{\partial y}\frac{\partial y}{\partial b} + \frac{\partial I}{\partial x}\frac{\partial x}{\partial b}$$

- State equations for fluid and structural system and their derivatives w.r.t design variables

Fluid: $R(w, y, x, b) = 0 \rightarrow \frac{dR}{db} = \frac{\partial R}{\partial b} + \frac{\partial R}{\partial w}\frac{\partial w}{\partial b} + \frac{\partial R}{\partial y}\frac{\partial y}{\partial b} + \frac{\partial R}{\partial x}\frac{\partial x}{\partial b} = 0$

Structure: $S(w, y, x, b) = 0 \rightarrow \frac{dS}{db} = \frac{\partial S}{\partial b} + \frac{\partial S}{\partial w}\frac{\partial w}{\partial b} + \frac{\partial S}{\partial y}\frac{\partial y}{\partial b} + \frac{\partial S}{\partial x}\frac{\partial x}{\partial b} = 0$

- As the residual derivatives are zero, they can be multiplied with an adjoint vector and added to the objective function derivative.

$$\frac{dI}{db} = \frac{\partial I}{\partial b} + \frac{\partial I}{\partial w}\frac{\partial w}{\partial b} + \frac{\partial I}{\partial y}\frac{\partial y}{\partial b} + \frac{\partial I}{\partial x}\frac{\partial x}{\partial b} + \lambda^T\left(\frac{\partial R}{\partial b} + \frac{\partial R}{\partial w}\frac{\partial w}{\partial b} + \frac{\partial R}{\partial y}\frac{\partial y}{\partial b} + \frac{\partial R}{\partial x}\frac{\partial x}{\partial b}\right) + \phi^T\left(\frac{\partial S}{\partial b} + \frac{\partial S}{\partial w}\frac{\partial w}{\partial b} + \frac{\partial S}{\partial y}\frac{\partial y}{\partial b} + \frac{\partial S}{\partial x}\frac{\partial x}{\partial b}\right)$$

= Zero

= Zero

$\boldsymbol{\lambda}$: adjoint vector for flow state
$\boldsymbol{\phi}$: adjoint vector for structural state

Adjoint based Sensitivity Analysis

- Coupled Adjoint Analysis
  - Rearranging the terms

$$\frac{dI}{db} = \frac{\partial I}{\partial b} + \lambda^T \frac{\partial R}{\partial b} + \phi^T \frac{\partial S}{\partial b}$$

$$+ \left[\frac{\partial I}{\partial w} + \lambda^T \frac{\partial R}{\partial w} + \phi^T \frac{\partial S}{\partial w}\right]\frac{\partial w}{\partial b} + \left[\left(\frac{\partial I}{\partial y} + \lambda^T \frac{\partial R}{\partial y} + \phi^T \frac{\partial S}{\partial y}\right)\right]\frac{\partial y}{\partial b}$$

= Zero = Zero

R : fluid residuals
S : Structural residuals
w: fluid state variables
y: structural state variables
x: mesh state variables
b: design variables

  - The arbitrary adjoint vector can be chosen to be independent of the derivative of state variables w.r.t. the design variables.
  - The adjoint equation to be solved

$$\begin{bmatrix} \frac{\partial R}{\partial w} & \frac{\partial R}{\partial y} \\ \frac{\partial S}{\partial w} & \frac{\partial S}{\partial y} \end{bmatrix}^T \begin{Bmatrix} \lambda \\ \phi \end{Bmatrix} = -\begin{bmatrix} \frac{\partial I}{\partial w} \\ \frac{\partial I}{\partial y} \end{bmatrix}$$

  - Total derivative of objective function w.r.t. design variables is

$$\frac{dI}{db} = \frac{\partial I}{\partial b} + \frac{\partial I}{\partial w}\frac{\partial w}{\partial b} + \frac{\partial I}{\partial y}\frac{\partial y}{\partial b} = \frac{\partial I}{\partial b} + \lambda^T \frac{\partial R}{\partial b} + \phi^T \frac{\partial S}{\partial b}$$

Contents

Derivation of Jacobian sub Matrices

- $\frac{\partial R}{\partial w}$ computed by differentiating the internal flux and boundary condition functions.

$$\begin{bmatrix} \frac{\partial R}{\partial w} & \frac{\partial R}{\partial y} \\ \frac{\partial S}{\partial w} & \frac{\partial S}{\partial y} \end{bmatrix}^T \begin{Bmatrix} \lambda \\ \phi \end{Bmatrix} = -\begin{bmatrix} \frac{\partial I}{\partial w} \\ \frac{\partial I}{\partial y} \end{bmatrix}$$

$\partial w \longrightarrow \partial R$

differentiate central flux and dissipation

differentiate the contribution of boundary condition

- $\frac{\partial S}{\partial y}$ calculated by differentiating structural equations of motion (S) with respect to structural state (y).

$$\begin{bmatrix} \frac{\partial R}{\partial w} & \frac{\partial R}{\partial y} \\ \frac{\partial S}{\partial w} & \frac{\partial S}{\partial y} \end{bmatrix}^T \begin{Bmatrix} \lambda \\ \phi \end{Bmatrix} = -\begin{bmatrix} \frac{\partial I}{\partial w} \\ \frac{\partial I}{\partial y} \end{bmatrix}$$

$$S = \omega D y - A y - \mathrm{B} f \qquad \frac{\partial S}{\partial y} = \omega D - A$$

- $\frac{\partial S}{\partial w}$ calculated by differentiating structural equations of motion (S) with respect to fluid state (w) by using chain rule.

$$\begin{bmatrix} \frac{\partial R}{\partial w} & \frac{\partial R}{\partial y} \\ \frac{\partial S}{\partial w} & \frac{\partial S}{\partial y} \end{bmatrix}^T \begin{Bmatrix} \lambda \\ \phi \end{Bmatrix} = -\begin{bmatrix} \frac{\partial I}{\partial w} \\ \frac{\partial I}{\partial y} \end{bmatrix}$$

$$\frac{\partial S}{\partial w} = \frac{\partial S}{\partial f}\frac{\partial f}{\partial p}\frac{\partial p}{\partial w} \qquad S = \omega D y - A y - \mathrm{B} f, \ \frac{\partial s}{\partial f} = -B$$

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$$\frac{\partial S}{\partial w} = \frac{\partial S}{\partial f}\frac{\partial f}{\partial p}\frac{\partial p}{\partial w}, \qquad \mathrm{p} = (\gamma - 1)\left(w_5 - \frac{1}{2w_1}({w_2}^2 + {w_3}^2 + {w_4}^2)\right), \; w = \begin{Bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{Bmatrix} = \begin{Bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho E \end{Bmatrix}$$

$$\delta \mathrm{p} = (\gamma - 1)\left(\delta w_5 + \frac{1}{2w_1^2}({w_2}^2 + {w_3}^2 + {w_4}^2)\delta w_1 - \frac{1}{w_1}(w_2 \delta w_2 + w_3 \delta w_3 + w_4 \delta w_4)\right)$$

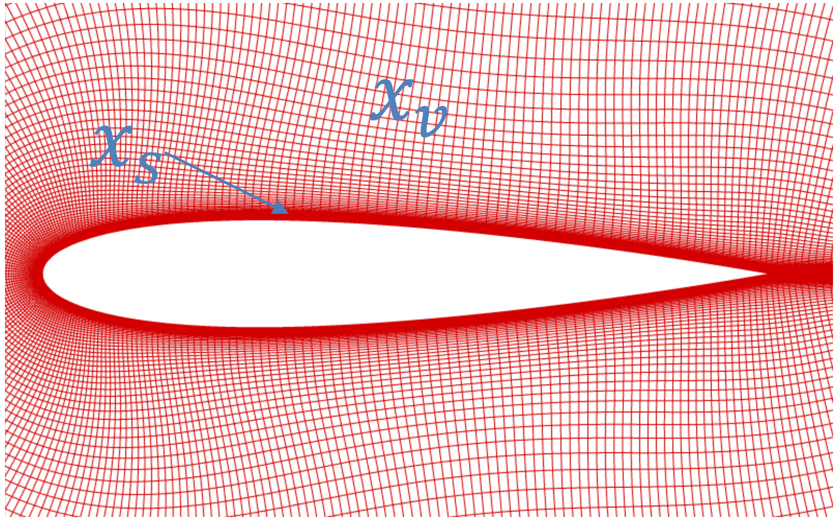
Derivation of Jacobian sub Matrices

- The fluid residual, ***R***, is not explicitly dependent on the structural state, ***y***, but through the wall boundary condition.

$$\begin{bmatrix} \frac{\partial R}{\partial w} & \frac{\partial R}{\partial y} \\ \frac{\partial S}{\partial w} & \frac{\partial S}{\partial y} \end{bmatrix}^T \begin{Bmatrix} \lambda \\ \phi \end{Bmatrix} = - \begin{bmatrix} \frac{\partial I}{\partial w} \\ \frac{\partial I}{\partial y} \end{bmatrix}$$

$$\frac{\partial R}{\partial y} = \frac{\partial R}{\partial s_x} \frac{\partial s_x}{\partial x_v} \frac{\partial x_v}{\partial x_s} \frac{\partial x_s}{\partial y}$$

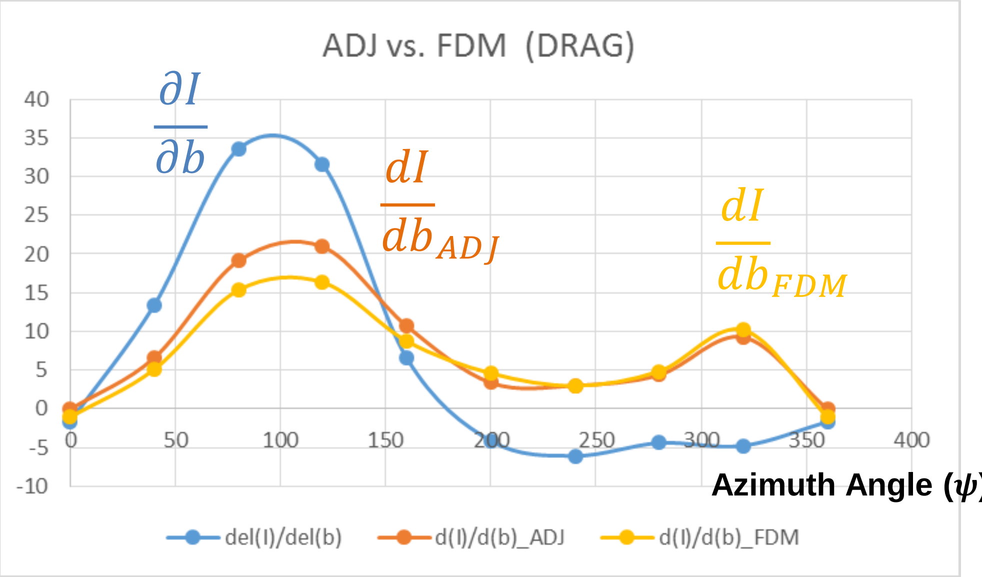
$$\partial y \longrightarrow \partial x_s \longrightarrow \partial x_v \longrightarrow \partial s_x \longrightarrow \partial R$$

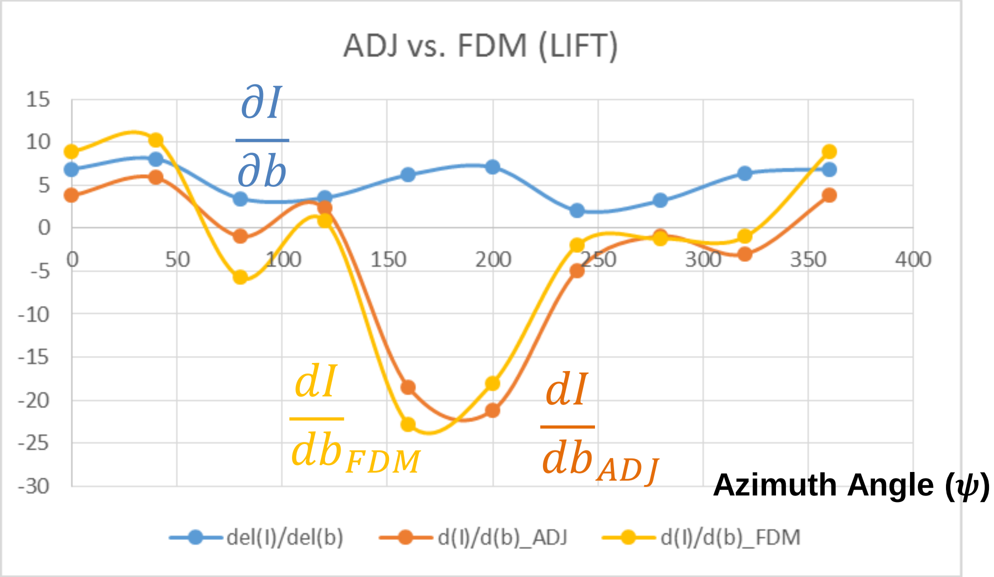


y structural state (deflection)
x_s surface mesh
x_v volume mesh
s_x mesh metrics
R aero residual

Sensitivity Analysis Results

- Sensitivity Analysis Results (Aero only)
  - Comparison with FDM (Finite Difference Method) analysis





$$\frac{dI}{db} = \frac{\partial I}{\partial b} + \boxed{\frac{\partial I}{\partial w}\frac{\partial w}{\partial b}} \qquad = \frac{\partial I}{\partial b} + \boxed{\lambda\frac{\partial R}{\partial b}}$$

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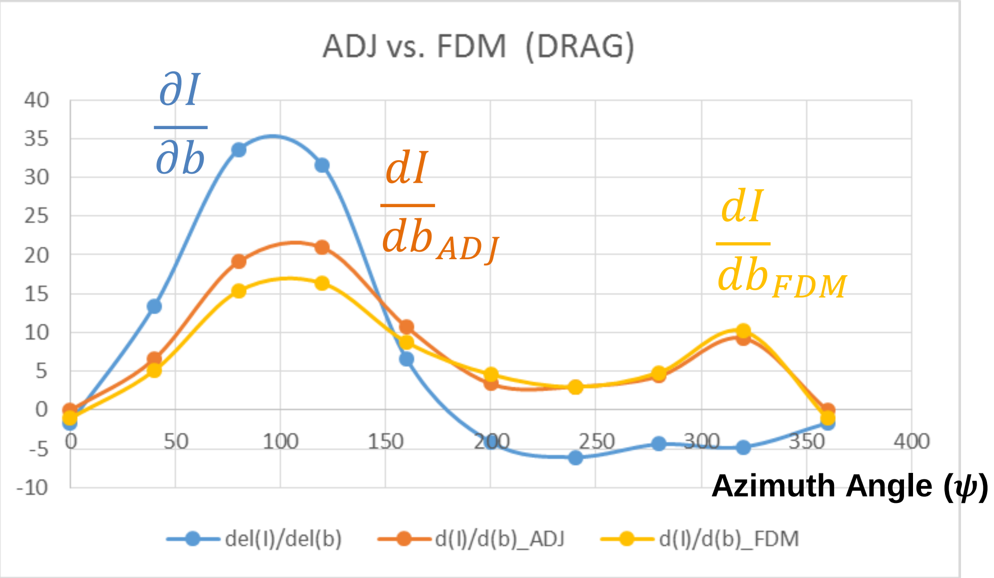
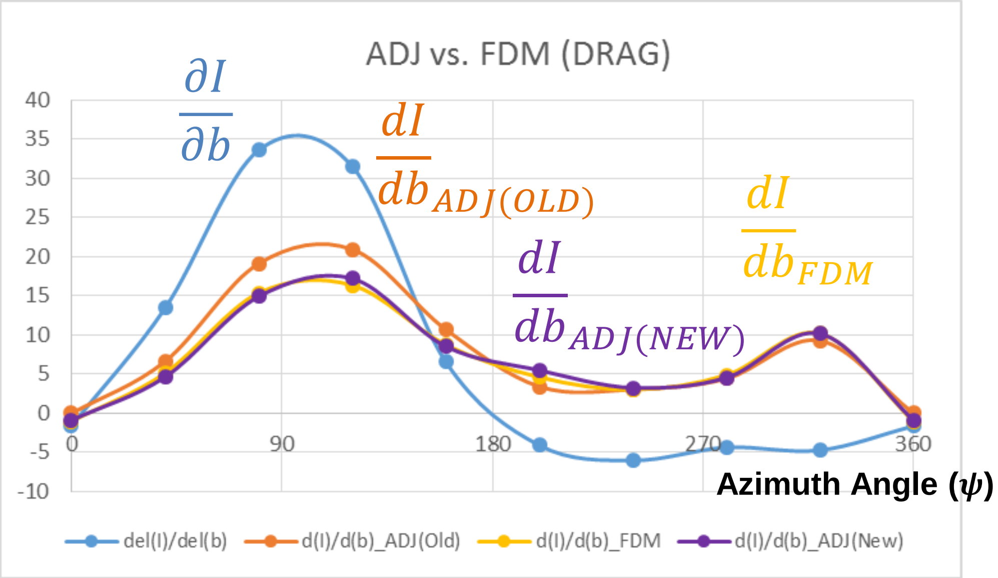
AMDL Aerospace Multi-Disciplinary Design Laboratory

Sensitivity Analysis Results

- Sensitivity Analysis Results (Aero only)
  - Comparison with FDM (Finite Difference Method) analysis **(updated after dissertation proposal)**





Sensitivity Analysis Results

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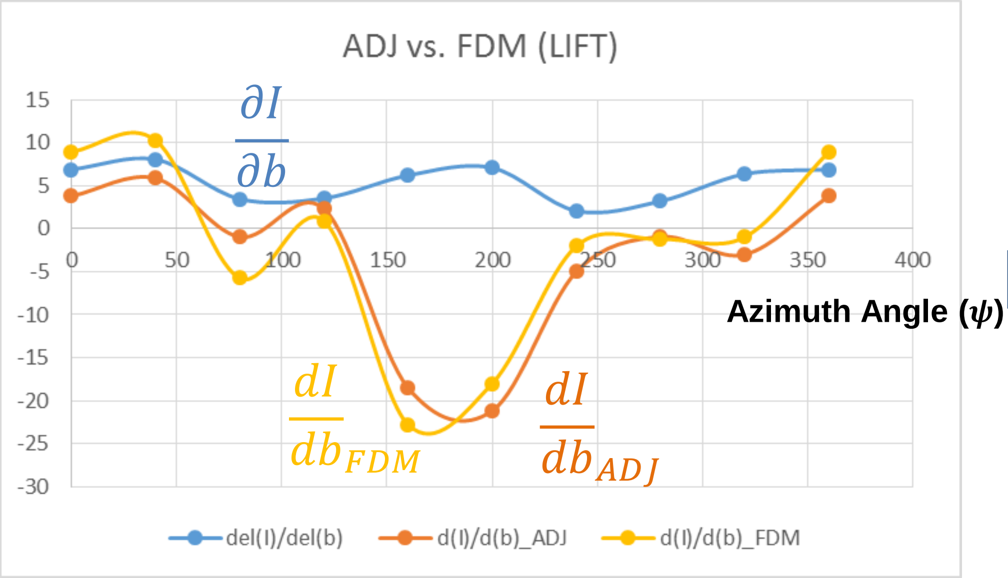
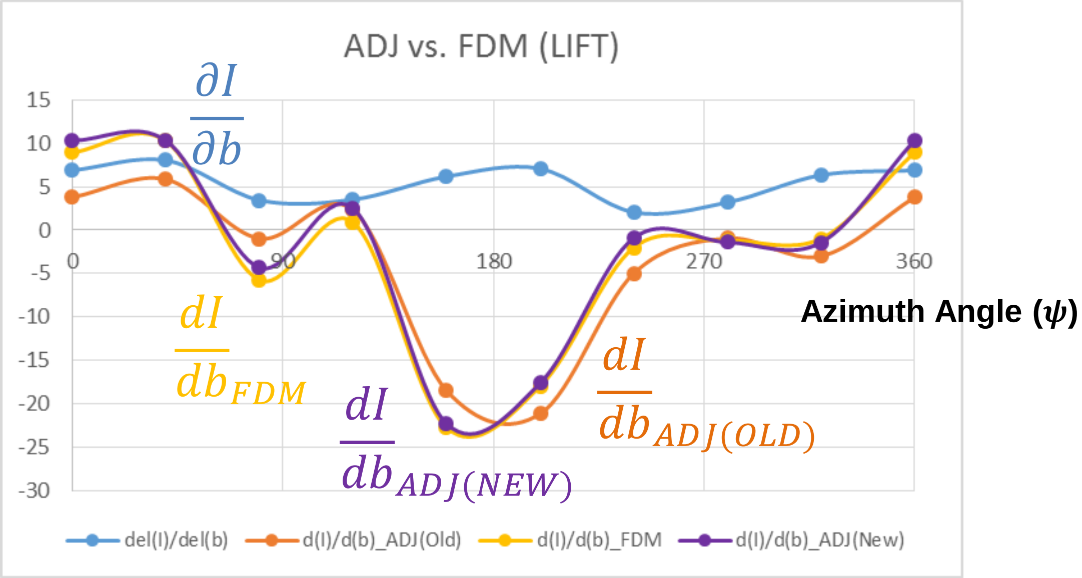




Sensitivity Analysis Results

- Finite Difference Method
  - Step Size Study (3 time instances)

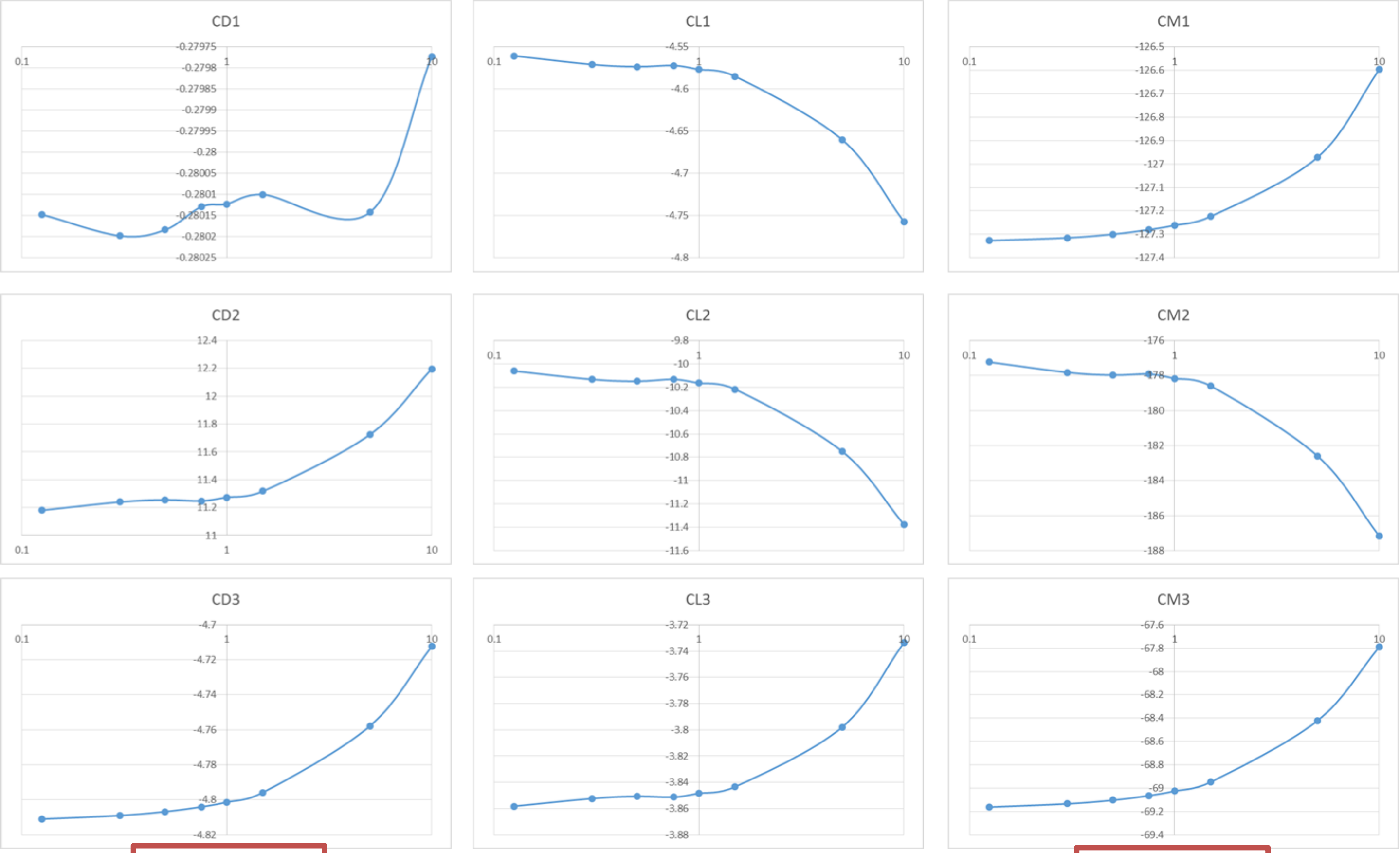
$$f'(x) \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

DRAG **LIFT** **MOMENT**

1st Time Instance

2nd Time Instance

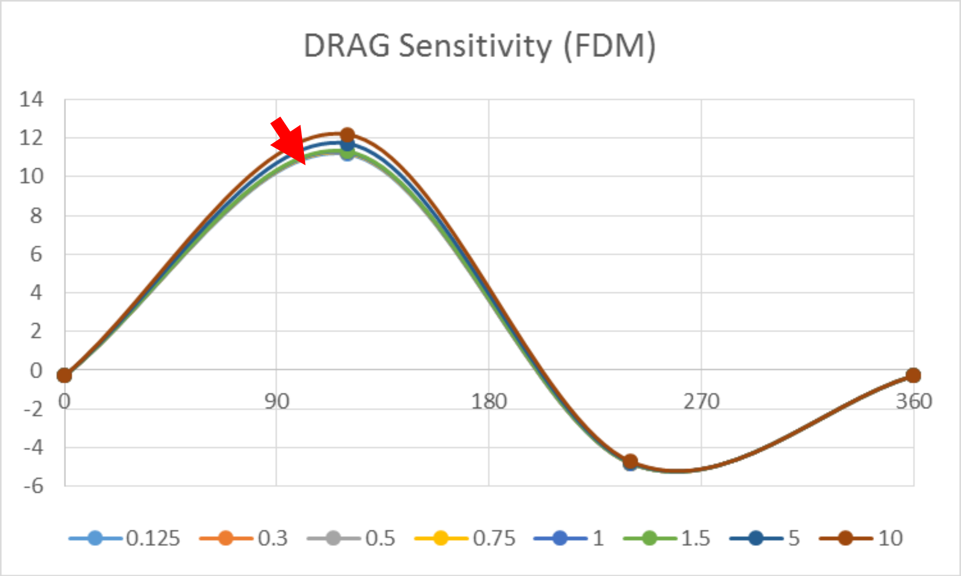
3rd Time Instance

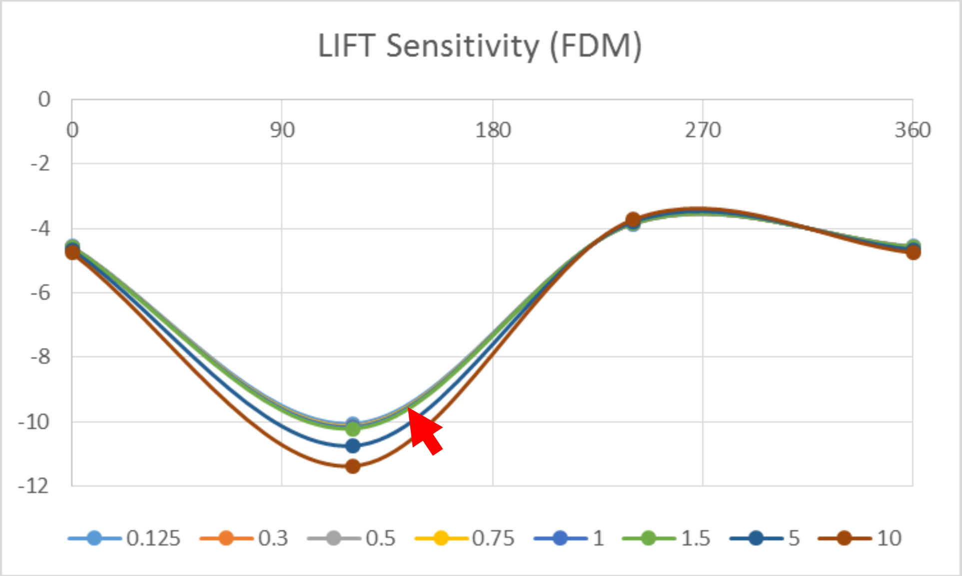


delP= 0.1 1.0 10.0 0.1 1.0 10.0

Sensitivity Analysis Results

- Finite Difference Method
  - Step Size Study (3 time instances)





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- Im, Dong Kyun, Hyunsoon Kim, and Seongim Choi. "Mapped Chebyshev Pseudo-Spectral Method for Dynamic Aero-Elastic P roblem of Limit Cycle Oscillation." *International Journal of Aeronautical and Space Sciences* 19, no. 2 (2018): 316-329. **(publis hed)**
- Prasad, Rachit, Hyunsoon Kim, Seongim Choi, and Seulgi Yi. "High fidelity prediction of flutter/LCO using time spectral method ." In *2018 AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, p. 0459. 2018. **(published)**
- Prasad, Rachit, Hyunsoon Kim, and Seongim Choi. "Flutter Related Design Optimization using the Time Spectral and Coupled Adjoint Method." In *2018 AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, p. 0101. 2018. **(p ublished)**
- Prasad, Rachit, Hyunsoon Kim, and Seongim Choi. "Adjoint based Finite Element Model Updating and Validation using Time D omain based FSI Analysis." In *58th AIAA/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, p. 1125 . 2017. **(published)**
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- “Coupled Adjoint-based Rotor Design using a Fluid Structure Interaction in Time Spectral Form” **(under preparation)**

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